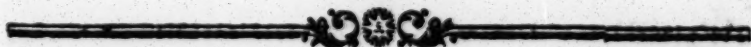




THE  
TYRO'S GUIDE  
TO  
ARITHMETIC  
AND  
MENSURATION.





THE  
TYRO'S GUIDE  
TO  
ARITHMETIC  
AND  
MENSURATION.

WITH  
An APPENDIX, containing a great  
many Questions both curious and useful.

The whole accommodated to the Capacity of Beginners,  
and designed for the use of Schools and Mechanics.

By WILLIAM PANTON, M. A.

Late Master of the Grammar-School of Canongate.

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EDINBURGH:  
PRINTED BY J. REID.  
Sold by the Bookfellers in town and country.

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SPECTATISSIMO VIRO,  
GULIELMO FORBESIO,  
EQUITI BARONETTO;

HOC ARITHMETICES AC MENSURA-  
TIONIS SPECIMEN,

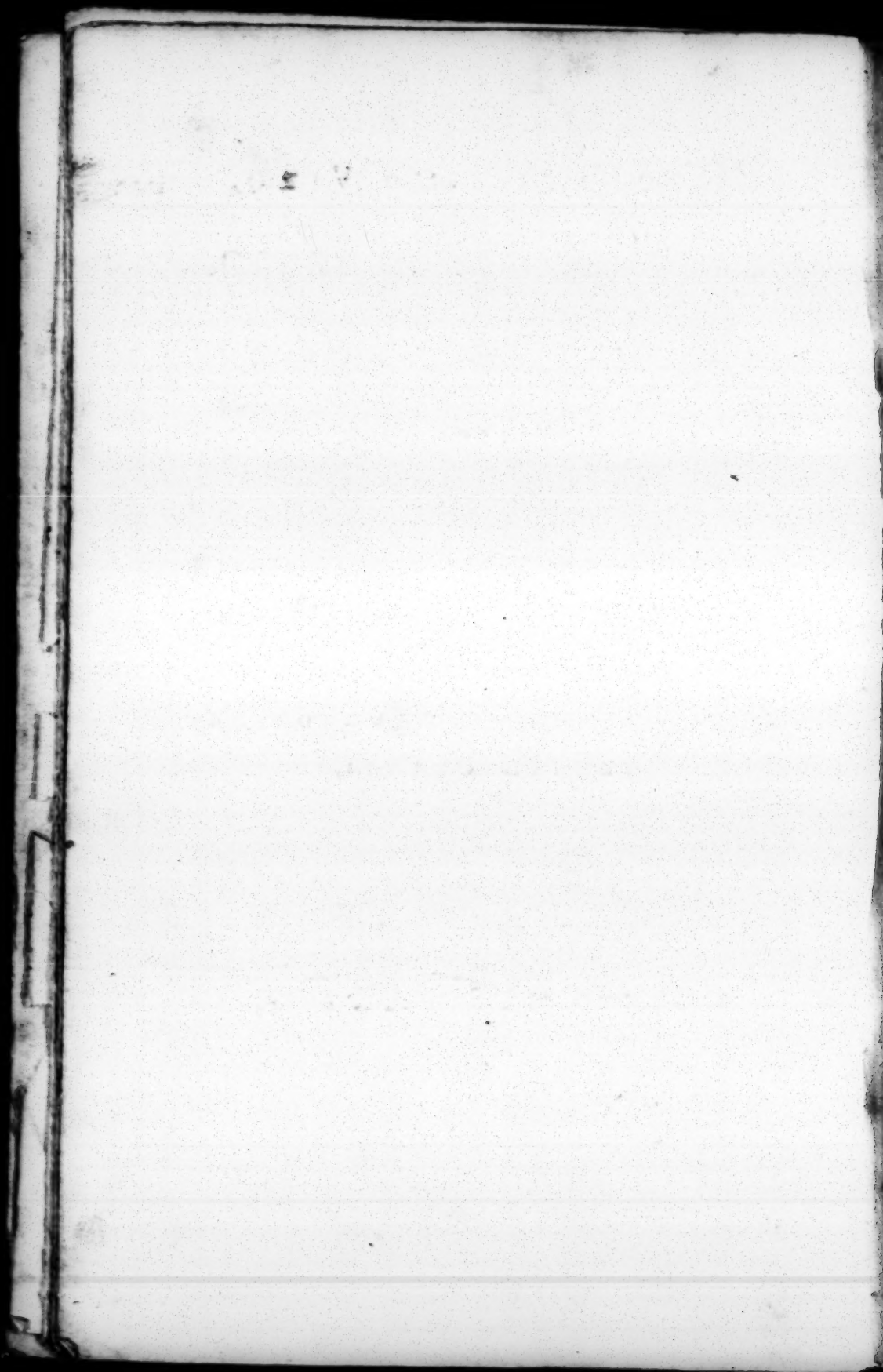
AD USUM EORUM, QUI IN HUIUS SCIENTIÆ  
STUDIUM INCUMBUNT, EXCOGITATUM,

BENEVOLENTIÆ ATQUE OBSER-  
VANTIÆ GRATIA,

D. D. C. Q.

GULIELMUS PANTON.

Apud Vicum<sup>u</sup> Canonicum,  
3tio Idus Quintilis, 1770.



T H E  
P R E F A C E.

**T**H E books already published on ARITHMETIC and MENSURATION are so very numerous, that the Public will doubtless be surpris'd to see a new one on that subject; because little can be said on it, but what has been already advanced by some one author or other. However, as some of these authors have treated the subject so superficially, as to neglect and overlook many useful rules, while others have spun it out to such a length, as to render it voluminous, and consequently difficult to be acquired, on account of the extravagant price; and lastly the incorrectness of many; these were powerful enough motives to the author to attempt a work of this nature. And in order that it might be the more acceptable to the Public, it was judg'd proper to have it revised by some Mathematicians and Teachers of Arithmetic in this place; whose approbation of the work entirely determin'd the author to make this public appearance. And if it meet with a favourable reception from the world, and be deem'd useful and advantageous to youth, the author will reckon his labour amply rewarded.

IN order to save invidious critics the trouble of disparaging this undertaking, or calling the author a sorry plagiarist, (an epithet not easily digested by many), he frankly acknowledges, and that without a blush, that he has been indebted to several who have trod the same path before, particularly for many of the questions in the Appendix ; which he has industriously lengthened out, because he judges it would be of no small advantage to youth, after having finished a regular course of Arithmetic, to study this appendix, as it gives a summary view of all the antecedent rules, and that with very little trouble.

— *Si quid novisti rectius istis,  
Candidus imperti ; si non, his utere mecum.*

H O R A T.

T H E

T H E  
T Y R O ' s G U I D E  
T O  
A R I T H M E T I C  
A N D  
M E N S U R A T I O N .

---

I N T R O D U C T I O N .

**I**T will not be improper, in this place, to premise the definitions of some terms both in Arithmetic and Mensuration, as also the signification of some marks or symbols made use of in the following sheets.

DEF. 1. Number is either unity or multitude.

2. An integer or whole number is 1, or any number of units.

3. A fraction is a part or parts of unity, according as the same is divided.

4. An aliquot part is a lesser number, which is contained a certain number of times in a greater, without a remainder; as 4 is an aliquot part of 8, 12, 16, 20, &c.

5. An

5. An aliquant number is a lesser one, which is contained in a greater a certain number of times with something remaining; as 4 is an aliquant part of 10, 14, 18, &c.

6. The common measure to any two or more numbers, is a number which can divide these numbers, without a remainder; thus 6 is a common measure to 18, 24, 36, &c.

7. The greatest common measure to any two or more numbers, is the greatest number that can divide the proposed numbers, without a remainder; thus 6 is the greatest common measure to 18 and 24.

8. A prime number is that which hath no measure but itself and unity; thus 3, 5, 7, 11, 13, &c. are prime numbers.

9. The multiplicand and multiplier go often by the name of factors, because being multiplied together they make the product.

10. A square, in geometry, is a figure consisting of four equal sides, and as many right angles, as fig. 1. but a square, in arithmetic, is that number produced by multiplying any number by itself; thus 16 is the square of 4.

11. A rectangle parallelogram or oblong is a figure consisting of four right angles, having its opposite sides equal; as fig. 2.

12. A rhombus is a figure having four equal sides, whereof the two opposite are equal and parallel, and may be represented by a diamond, or square out of its true position; as fig. 3.

13. A

13. A rhomboides has four equal sides, where-  
of the two opposite are equal and parallel, and  
the opposite angles are equal, being a parallelo-  
gram out of its true position ; as fig. 4.

14. A triangle has three sides and three  
angles, which are equal to two right angles, each  
90 degrees ; as fig. 5.

15. An angle is the inclination of two lines,  
meeting one another in a point, so that the recti-  
lineal angles, according to the greater or lesser  
degree of inclination, are either right, acute, or  
obtuse.

16. A right angle is that formed between two  
lines, one of which stands upright or perpendi-  
cularly on the other, inclining no more one way  
than it does the other, and is 90 degrees.

17. An acute angle is less than a right one, or  
less than 90 degrees.

18. An obtuse angle is greater than a right  
one, or more than 90 degrees.

19. A trapezium is a figure consisting of four  
sides and four angles, which are generally neither  
parallel nor equal ; as fig. 6.

20. A polygon is a figure consisting of more  
than four sides ; and is either regular, as fig. 8.  
which is called a pentagon or irregular, as fig.  
7.

21. A circle is a plane figure, comprehended  
by a single curve line, called its circumference, to  
which right lines, or radii, drawn from a point  
in

in the middle, called the centre, are equal to each other; as fig. 9.

22. The circumference, in a general sense, denotes the line or lines bounding a plain figure. However, in a more limited sense, it is generally used for the curve line that bounds a circle, and otherwise called a periphery; the boundary of a right-lined figure being expressed by the term perimeter.

23. The diameter is the right line passing thro' the centre of a circle, and terminated at each side by the circumference thereof.

24. The conjugate diameter, or axis of an ellipsis, is the shortest of the two diameters, or that bisecting the other, which is called the transverse, and is the longest of the two diameters.

25. A diagonal is a right line drawn across a quadrilateral figure, from one angle to another, by some called a diameter.

26. An ellipsis is a curved line returning into itself, and produced by the section of a cone by a plane cutting both its sides, but not parallel to the base; as fig. 10.

27. A cube, in geometry, is a solid body, consisting of six equal square sides; as a die, or fig. 11.

A cube, in arithmetic, is that number which is produced by the multiplication of a square number by its root.

28. A parallelopipedon is a regular solid, comprehended

comprehended under six parallelograms, the opposite ones whereof are similar, parallel, and equal; as fig. 12.

29. A pyramid is a solid, which decreases gradually from the base till it comes to a point, which is called the vertex; and there are different kinds of pyramids according to the figure of the base; hence they are said to be triangular, parallelogrammic, or circular, as fig. 13. 14. 16.

30. A cylinder is a solid body, supposed to be generated by the rotation of a parallelogram about one of its sides, and resembles a rolling stone in a garden; as fig. 15.

31. A prism is a solid, contained under several planes; two of which being opposite, viz. the two ends, are called the bases, and these are parallel and equal; and the other planes are parallelograms, in which a right line may every where be applied from base to base.

Prisms are either triangular, multiangular, circular, or elliptical, &c. according to the figure of the base; thus a cube, a parallelopipedon, and a cylinder are prisms.

32. A cone is a solid figure, having a circle for its base, and its top terminated in a point or vertex; as fig. 17.

33. A sphere is a solid body, formed by the rotation of a circle about its own axis; as fig. 18.

34. A spheroid is a solid, formed by the rotation of the semi-ellipsis about its transverse diameter, which is called the spheroid's axis: this  
body

- body much resembles the shape of an egg; as fig. 19.

35. A frustum is a part of a solid body separated from the rest; and hence the frustum of a cone or pyramid, is the part that remains, when the top is cut off by a plane parallel to the base.

*Explanation of the marks or symbols.*

The sign  $+$  (plus or more) is the sign of addition.

The sign  $-$  (minus or less) is the sign of subtraction.

The sign  $\times$  (multiplied by) is the sign of multiplication.

The sign  $\div$  (divided by) is the sign of division.

The sign  $=$  (equal to) is the sign of equality.

The sign  $:$  ::  $:$  is the sign of proportion; and read thus, as 3 is to 6, so is 8 to 16.

There are several other mathematical signs in use; but as they do not occur in this work, it is unnecessary to exhibit or explain them.

# ARITHMETIC.

**A**RITHMETIC is said to have been invented at first by the Indians, and afterwards made more general by the Arabians. The utility and absolute necessity of which will be always fully asserted by all mechanics, mathematicians, and those of the mercantile profession. It is an art or science that teaches us the dexterous handling of figures, comprised under the nine digits and a cipher. The use and practice whereof depends upon a thorough knowledge of the five following rules, *viz.* NOTATION, ADDITION, SUBTRACTION, MULTIPLICATION, and DIVISION.

To treat of these five fundamental rules in the most perspicuous and concise manner, will be attempted in this treatise; and that upon a plan somewhat new. And therefore we begin with the first, *viz.*

## NOTATION.

**N**OTATION, by which we learn to give a just value to, or place aright any number of figures propounded; in order to which, regard must be had to the following table, which we shall place immediately after inserting the nine digits, and the Roman characters which are most rarely to be met with.

The nine digits are these, 1, 2, 3, 4, 5, 6, 7, 8, 9.

The old Roman numbers are,

1000 M or CIO  
2000 CIO. CIO. or M.M.  
3000 CIO. CIO. CIO. or M. M. M  
5000 ICO.

10000 CCICD  
 50000 ICDD  
 100000 CCCICDD or CM  
 500000 ICDD

## N O T A T I O N - T A B L E.

5	Units	9
50	Tens	98
500	Hundreds	987
5000	Thousands	9876
50000	X of thousands	98765
500000	C of thousands	987654
5000000	Millions	9876543
50000000	X of millions	98765432
500000000	C of millions	987654321

If you add a cipher or ciphers to, or subtract them from, or place them on the left hand of any whole number, they can neither increase nor diminish that number : but if you place them on the right hand of any whole number, they increase its value in a tenfold proportion ; as you will observe from the above table : and that every figure hath two values in said table, one in itself, and the other from the place it stands in ; for a figure when standing alone, or in the units place of any number, has its simple value ; but a figure in the second place, has ten times the value it would have, were it in the first place, or place of units ; and a figure in the third place has ten times the value it would have, were it in the second place ; and so each place has ten times the value of that immediately preceding it.

It may be observed, that the order of places is reckoned from the right hand to the left ; but (like that of letters or words) numbers are to be read from the left hand to the right, and so many figures

figures as are placed together without any point, comma, line, or other note of distinction between them, are all but one sum, and must be read as such.

In reading any number larger than the last in the preceding table, which consists only of nine figures, consider, first, that every third figure from the place of units, bears the name of hundreds; and so let every third figure be pointed (as in the following number) below the line; and again, observe, that the figure on the left hand of each second hundred place is millions, billions, trillions, quadrillions, &c. and let these be marked with a point above the line.

quadrillions trillions billions millions units  
 64,532,167,891,234,892,367,145,392,645,782

Read thus, sixty-four quintillions, five hundred and thirty two thousand one hundred sixty-seven quadrillions, eight hundred ninety-one thousand two hundred thirty-four trillions, eight hundred ninety-two thousand three hundred sixty-seven billions, one hundred forty-five thousand three hundred ninety-two millions, six hundred forty-five thousand seven hundred and eighty-two.

In placing down any number arithmetically, write down the figures in the same order their values are expressed, beginning at the left hand, and writing towards the right: and if in pronouncing the number, any places are omitted, these must be supplied with ciphers.

Write down Seven hundred sixty-four millions five hundred sixty-eight thousand nine hundred and thirty four. *Ans.* 764568934.

Write down Nine millions five hundred and seven. *Ans.* 9000507.

Write down One million wanting one. *Ans.* 999999.

## 16 ADDITION of INTEGERS.

Write down Eleven millions eleven thousand eleven hundred and eleven. *Ans.* 11012111.

Write down Eighteen millions eighteen thousand eighteen hundred and eighteen. *Ans.* 18018818.

## ADDITION of INTEGERS.

**A**DDITION is that rule by which several numbers or quantities are collected and put together; and that quantity which arises or results from thence, is called the sum or total amount of these quantities.

Addition is of one or different denominations.

Addition of one denomination is, when the several quantities given to be added are all of one name or species; *i. e.* all pounds, acres, miles, feet, &c.

The numbers to be added, must be placed in such order under one another, (it matters not which is uppermost, the greatest or least), that units may stand under units, tens under tens, hundreds under hundreds, thousands under thousands, &c.

**RULE.** Always begin your addition at the place of units, adding together all the figures that stand in that place or column; and if their sum be under ten, set it down below a line drawn under the figures proposed to be added, in the place of units; but if it amount to ten or any number of tens precisely, set down a cipher; and if above ten or tens, set down the excess, and carry one for every ten to the undermost figure of the next column; and thus proceed to the last column, setting down the whole amount thereof, and so you will have the sum total.

E X.

# SUBTRACTION of INTEGERS. 17

## EXAMPLES.

£.	Cwt.	Months.	Years.
27	345	2345	56789
35	678	6789	12345
47	912	1234	67891
35	340	5678	2345
41	567	9123	6789
36	891	456	1234
56	203	789	567
37	456	123	891
17	789	456	234
—	—	—	—
331	5181	26993	149085

The common method used to prove Addition, is by scoring off the uppermost line, and finding the total amount of the rest, which when added to the line formerly cut off, will be equal to the first, if right; otherwise, it is wrong.

But after adding your several columns upward, if you begin again at the last, and add them downwards; then if right, your total sums will be equal. This method is accounted more expeditious, and as little subject to error as the other.

We should now proceed to Addition of various denominations, according to the plan of most authors upon this subject; but we judge it more eligible to adopt that method which postpones this kind of Addition till Division once is learned.

## SUBTRACTION of INTEGERS.

**B**Y Subtraction we find the difference or remainder of any two numbers, by taking the lesser (called the subtrahend) from the greater (called the minuend); therefore, in placing your numbers

## 18 SUBTRACTION of INTEGERS.

numbers to be subtracted, care must be taken not only to place the greatest sum uppermost, but also units must stand under units, and tens under tens, &c.

**RULE.** When the undermost figure is greater than that immediately above it, borrow ten to the uppermost, and then subtract; and for the ten thus borrowed, always remember to add one to the next lowermost figure; and thus proceed from the right hand till you come to the last on the left.

### EXAMPLES.

	£.	Cwt.	Yards.
I borrowed	76593	321045	5403214
I paid	<u>59638</u>	<u>245697</u>	<u>4637426</u>
Remains	<u>16955</u>	<u>75348</u>	<u>765788</u>
Proof	76593	321045	5403214

The proof of Subtraction is known, by adding the remainder to the undermost line, the sum whereof will be equal to the higher if right; otherwise, your work is wrong.

What sum added to 391 l. will make 1000 l.?

$$\begin{array}{r}
 \text{£.} \\
 1000 \\
 391 \\
 \hline
 \text{Ans. } 609 \\
 \hline
 1000
 \end{array}$$

How

## M U L T I P L I C A T I O N : 19

How old is the man that was born *anno* 1692, this being the year 1771? *Ans.* 79.

How long is it since the fire of London, which happened *anno* 1666? *Ans.* 105.

From 90 take 30, from 40 take ten,  
Subtract 6 from 60, and what remains then?  
*Ans.* 144.

In fifteen hundred 92 there did a noble Prince;  
How many years is it ago, that is, how many since?  
*Ans.* 179.

What is the difference between twice eight and twenty and twice twenty-eight? *Ans.* 20.

## M U L T I P L I C A T I O N of I N T E G E R S.

**I**N Multiplication we always have two numbers given, commonly called factors, *viz.* the multiplicand and multiplier: the first is the number to be multiplied, and the other is that by which we multiply; the number arising from these two is called the product, which contains the multiplicand as often as the multiplier does unity. But before we proceed to give examples, it will not be improper to affix, for the use of the learner,

# 20 MULTIPLICATION

## A TABLE of MULTIPLICATION.

2 times 2 is 4	3 times 7 is 21	5 times 5 is 25	7 times 7 is 49
3 6	8 24	6 30	8 56
4 8	9 27	7 35	9 63
5 10	12 36	8 40	12 84
6 12		9 45	
7 14	4 times 4 is 16	12 60	8 times 8 is 64
8 16	5 20		9 72
9 18	6 24	6 times 6 is 36	12 96
12 24	7 28	7 42	
3 times 3 is 9	8 32	8 48	9 times 9 is 81
4 12	9 36	9 54	12 108
5 15	12 48	12 72	
6 18			12 times 12 is 144

## EXAMPLES.

Multiply	4693	and	68349	Multiplicand
by	6		9	Multiplier
	<u>28158</u>		<u>615141</u>	Product

When your multiplier consists of two or more figures, care must be taken to place the figure arising from the multiplication of your second figure in the multiplier into the first of the multiplicand, below the said second figure, or, which is the

the same, in the place of tens ; and this you must observe to do with all the rest, viz. the third figure in the place of hundreds, and so on.

EXAMPLES.

Multiply 478342 by 468 <hr/> 3826736 2870052 1913368 <hr/> 223864056	2134678912 1236 <hr/> 12808073472 6404036736 4269357824 2134678912 <hr/> 2638463135232
---	--

If you have ciphers either in your multiplicand or multiplier, you need only multiply your significant figures, and then annex your ciphers.

EXAMPLES.

Multiply 8503400 by 579 <hr/> 76530600 595238 425170 <hr/> 4923468600	46815 6000 <hr/> 280890000
--	----------------------------------

If you have ciphers interspersed with your significant figures in the multiplier, as in the subsequent example, multiply only the significant figures,

## 22 M U L T I P L I C A T I O N

gures, placing the right-hand figure of each product below the multiplying figure.

$$\begin{array}{r}
 786804768 \\
 806009 \\
 \hline
 7081242912 \\
 4720828608 \\
 6294438144 \\
 \hline
 634171724250912
 \end{array}$$

Multiply 271047	7092851	2705197
by 32104	37154	207519
<hr/>	<hr/>	<hr/>

$$8701692888 \quad 263527786054 \quad 561379776243$$

Multiply 123456789	987654321
by 987654321	123456789
<hr/>	<hr/>

$$121932631112635269 \quad 121932631112635269$$

There are many compendious ways of working multiplication of integers, and we shall here give some examples of these which appear most necessary and useful.

1. To multiply by 12, 13, 14, &c. is no more than to multiply by 2, 3, 4, &c. and as you multiply, to add that figure of the multiplicand which stands on the right hand, as will appear by the following

### E X A M P L E S.

Multiply 12345	6729004	54321
by 13	19	16
<hr/>	<hr/>	<hr/>
160485	127851076	869136

2. To multiply by 112, 113, &c. at one operation :

to do which, you must multiply by 2, 3, &c. and as you multiply, add those two figures of your multiplicand which stand on the right.

EXAMPLES.

Multiply 654321	4246	642341
by 115	111	119
<hr style="width: 100%;"/>	<hr style="width: 100%;"/>	<hr style="width: 100%;"/>
75246915	471306	76438579

3. To multiply by 101, 102, &c. is no more than to multiply by 1, 2, &c. and as you multiply, add that figure of your multiplicand that standeth next your right hand except one, as is obvious from the sublequent

EXAMPLES.

Multiply 4321	427005	604150
by 106	101	109
<hr style="width: 100%;"/>	<hr style="width: 100%;"/>	<hr style="width: 100%;"/>
458026	43127505	65852350

4. To multiply any quantity by any number of nines, as 99, or 999: RULE. Annex to the right-hand figure of your multiplicand as many ciphers as your multiplier has 9's, and then subtract your multiplicand.

As  $8756 \times 99 = 866844$  and

875600
<hr style="width: 100%;"/>
8756
<hr style="width: 100%;"/>
866844

5. To multiply any number consisting entirely of nines by itself:

RULE. Set 1 in the place of units, then as many ciphers, except one, as there are nines in the multiplicand,

## 24 MULTIPLICATION.

multiplicand, then 8, and on the left hand as many nines as there are ciphers on its right hand.  
Thus

$$\begin{array}{rcl}
 99 \times 99 = 9801 & & \\
 999 \times 999 = 998001 & \left. \begin{array}{l} \text{for according to rule} \\ 4. \end{array} \right\} & \begin{array}{r} 999000 \\ 999 \\ \hline 998001 \end{array}
 \end{array}$$

The method for proving Multiplication by several authors, is by ejecting the 9's in the multiplicand, multiplier, and product: thus:

$$\begin{array}{r}
 786 \\
 \underline{47} \\
 5502 \\
 3144 \\
 \hline
 36942
 \end{array}
 \qquad
 \begin{array}{r}
 3 \quad 2 \\
 \quad X \\
 6 \quad 6
 \end{array}$$

After throwing out the 9's in the multiplicand, 3 remains, which I place on one point of the X, as above; then throwing out the 9's in the multiplier, 2 remains, and this I place on the opposite point; next multiplying 3 by 2, 6 is produced, which I place under the third point; and having ejected all the nines in the product, 6 must remain, if right; and this I place opposite to the other. But this method being subject to error, the best and surest way to prove multiplication is by division; by making your product your dividend, and the multiplier the divisor, and then the quotient will equal the multiplicand.

D I-

## DIVISION OF INTEGERS.

**I**N Division are three principal parts to be taken notice of.

1. The dividend, or number to be divided.
2. The divisor, or number by which we divide.
3. The quotient, or number proceeding from the other two. Sometimes there occurs a fourth, called the remainder.

In Division it holds,

As the divisor : to an unit : the dividend : the quotient.

Division is either single or compound.

Single division is, when the divisor is but one figure, and the dividend but two at most.

Compound division is, when the dividend consists of many places, and the divisor of one or more places.

**RULE.** Place your divisor before your dividend, with a curved or straight line betwixt them, and another after your dividend, to contain your quot; then distinguish, with a point, so many places of your dividend towards your left hand, as are equal or next exceeding your divisor; and then asking how oft your divisor is contained in the said sum? the answer must be placed in your quot, on the right hand of the dividend; then multiply your divisor by said figure, setting the

† C

product

## 26 DIVISION of INTEGERS.

product thereof under your aforesaid distinguished sum; and, after drawing a line below them, subtract the lower from the higher, and to the remainder bring down the next figure in the dividend, after having pointed the same, (in order to avoid taking down one figure twice, or any figure out of its due order), with which proceed as before, and so on till you have pointed and exhausted all your dividend.

### EXAMPLES.

$$6) 345678 \text{ (57613)}$$

30

45. To be performed rather thus

42

$$6) 345678$$

57613

36

36

7

6

18

18

This last method every learner ought to practise, as being more expeditious; and, by repeated practice, will become equally easy when the divisor is 12, or any lesser number.

Divide

# DIVISION of INTEGERS. 27

Divide 76453281 by 2468

$$\begin{array}{r}
 2468 \overline{) 76453281} \quad (30977 \frac{1041}{2468} \\
 \underline{7404} \phantom{00} \\
 24132 \\
 \underline{22212} \phantom{00} \\
 19208 \\
 \underline{17276} \phantom{00} \\
 19321 \\
 \underline{17276} \phantom{00} \\
 2045 \\
 \underline{2468}
 \end{array}$$

From the operation of the preceding question, it appears, that the whole steps in division are contained in the following monastich.

1                      2                      3                      4

*Dic quot, multiplica, subduc, transferque sequentem.*

First ask, how often the divisor is got ?

The answer gives the figure in the quot ;

Subtract the product of these two, and then

Bring down the next, and ask how oft again ?

# 26 DIVISION of INTEGERS.

EXAMPLES for practice.

$$35)7210473(206013$$

$$473)2104721(4449$$

$$275)3720147(13527$$

$$3701)72109521(19483$$

$$3576)72104725(20163$$

$$2510)63210476(25183$$

$$25204)321047217(12737$$

$$31709)521047321(16432$$

$$725014)72527103521(100035$$

$$2701234)7210472532(2669$$

$$210472)352107913214(1672944$$

$$3721071)21071921473(5662$$

The most infallible proof of division is by multiplication. **RULE.** Multiply your quotient by your divisor, and the product will just answer the dividend; for 9 divided by 3 gives 3, which multiplied by 3 gives 9.

## TABLES of MONEY, &c.

**B**Efore we proceed to treat of Addition, &c. of different denominations, the following tables of

**T A B L E S** of MONEY, WEIGHTS, &c. 29  
of coin, weight, and measure, must necessarily be  
inserted.

*Money.*

4 farthings = 1 penny	}	Farthings by $\frac{1}{4}$ = 1 farth.
12 pence = 1 shilling		some mark- $\frac{1}{2}$ = 2 farth.
20 shillings = 1 pound		ed thus : $\frac{1}{4}$ = 3 farth.

Note, That a noble is 6 s. 8 d. and 2 nobles make  
1 merk, or 13 s. 4 d.

*Scots Troy Weight.*

2 half-drops = 1 drop  
16 drops = 1 ounce  
16 ounces = 1 pound  
16 pounds = 1 stone  
25 pounds = 1 quarter  
4 quarters = 1 hundred

*English Troy Weight.*

24 grains = 1 penny-wt.  
20 penny-wt. = 1 ounce  
12 ounces = 1 pound

N. B. By Scots and English Troy weight are  
weighed jewels, gold, silver, bread, corn, and all  
liquors. The proportion of the Scots to the Eng-  
lish weight is as 100 to 108  $\frac{1}{4}$ ; that is, 100 lb. Scots  
= 108 lb. 9 oz. English.

*Scots Tron Weight.*

36 grains = 1 drop  
16 drops = 1 ounce  
16 ounces = 1 pound  
16 pounds = 1 stone  
25 pounds = 1 quarter  
4 quarters = 1 hundred

*English Avoirdupoise.*

16 drams = 1 ounce  
16 ounces = 1 pound  
14 pounds = 1 stone  
28 pounds = 1 quarter  
8 stones or 4 qrs = 1 hun.  
20 hundred wt. = 1 ton

N. B. In Scotland and England, butter, cheese,  
all grocery goods, are weighed by Tron and A-  
voirdupoise

# 30 TABLES of WEIGHTS & MEASURE.

voirdupoise respectively. And the proportion betwixt Troy and Avoirdupoise weight, by a very nice experiment, is found to be, 1 lb. Avoirdupoise equal to 14 oz. 11 pwt.  $15\frac{1}{2}$  grains Troy.

## *Apothecaries Weight.*

20 grains = 1 scruple  
3 scruples = 1 dram  
18 drams = 1 ounce  
12 ounces = 1 pound

Apothecaries compound their drugs by this weight, but buy and sell by Avoirdupoise.

## *Wool Weight.*

7 pounds = 1 clove  
2 cloves = 1 stone  
2 stones = 1 todd  
 $6\frac{1}{2}$  todts = 1 wey  
2 weys = 1 sack  
12 sacks = 1 last

This weight is entirely English, and differs only in the denomination of its parts from Avoirdupoise.

## *Scots Liquid Measure.*

6.44375 solid in. = 1 gill  
4 gills = 1 mutchkin  
2 mutch. = 1 chopin  
2 chopins = 1 pint  
2 pints = 1 quart  
4 quarts = 1 gallon  
16 gallons = 1 hoghead  
2 hogheads = 1 pipe  
2 pipes = 1 tun

## *English Liquid Measure.*

$35\frac{1}{4}$  solid in. = 1 pint wat.  
 $28\frac{1}{4}$  solid in. = 1 pint ale  
2 pints = 1 quart  
2 quarts = 1 pottle  
2 pottles = 1 gallon  
63 gallons = 1 hoghead  
42 gallons = 1 tierce  
2 hoghead = 1 pipe  
2 pipes = 3 tun

Note, The standard Scots pint, kept by the Dean of Guild of Edinburgh, contains exactly 103.1 solid inches. Three Scots pounds of the water of Leith, is the standard of the above pint.

Two pints and half a gill Scots is equal to 3 English quarts. The English ale quart, kept as a standard

# TABLES of MEASURE. 31

Standard at Edinburgh, contains  $70\frac{1}{2}$  solid inches.  
21 Scots pints, abating 24 solid inches, are exactly equal to the English bushel kept at Edinburgh.

## Scots dry Measure.

103.1 solid inch.=1 pint  
5.31 pints =1 peck  
21.25 pints=1 fir. wheat  
31 pints =1 fir. oats  
4 firlots =1 boll  
4 bolls =1 quarter  
4 quarters=1 chalder

## English dry Measure.

333.6 solid inch.=1 pint  
2 pints =1 quart  
2 quarts=1 pottle  
2 pottles=1 gallon  
2 gallons=1 peck  
4 pecks =1 coom  
2 cooms =1 quarter  
4 quarters=1 chalder

N. B. 3.1 Winchester bolls are a Scots boll of oats and malt, and  $2\frac{1}{8}$  Winchester bolls are a Scots boll of wheat, rye, pease, and meal.

## Scots long Measure.

3 barley corns=1 inch  
12 inches =1 foot  
3 feet 1.05 inch.=1 ell  
6 ells =1 fall  
40 falls =1 fur.  
8 furlongs =1 mile  
62 miles =1 degree

N. B. 11 measured Scots miles are equal to 12 measured English d°.

## English long Measure.

3 barley corns=1 inch  
12 inches =1 foot  
3 feet =1 yard  
 $5\frac{1}{2}$  yards =1 pole  
40 poles =1 fur.  
8 furlongs =1 mile  
3 miles =1 league  
 $69\frac{1}{2}$  yards =1 degr.  
360 degrees =1 circum.

The original of long measure is from a corn of barley, whereof 3 taken out of the middle of the ear, and well dried, make 1 inch; and therefore 1 barley-corn is the least measure, but not used in accounts.

Scots

# 32 TABLES of MEASURE & TIME.

<i>Scots Cloth Measure.</i>	<i>English Cloth Measure.</i>
2 half-nails=1 nail	2 half-nails=1 nail
4 nails =1 quarter	4 nails =1 quarter
4 quarters =1 ell	4 quarters=1 yard
	5 quarters=1 ell

Note, The old iron Scots ell, kept as the standard by the Dean of Guild at Edinburgh, is equal to  $\frac{27}{7}$  parts of the English ell: the said iron ell contains 37.05 English inches. The English brass yard, which was sent down to Edinburgh at the union, contains 35.95 English inches.

<i>Scots Land Measure.</i>	<i>English Land Measure.</i>
36 square ells=1 fall	30 $\frac{1}{2}$ square yar.=1 pole
40 square falls=1 rood	40 square poles=1 rood
4 roods =1 acre	4 roods =1 acre

Note, The Scots acre is to the English d<sup>o</sup> as 6 to 5.

A chain for measuring so as to find its content in Scots acres, the same way as in English acres by Gunter's chain, ought to contain 24 Scots ells of 37 inches to the ell, and should be divided into 100 links, each link containing 8.88 inches.

## Time.

60 seconds	=1 minute	
60 minutes	=1 hour	
24 hours	=1 day	
7 days	=1 week	
13 months and }		
1 day }	=1 year	or 365 days=1 year

N. B.

N. B. The true tropical year consists of 365 days 5 hours 48 minutes 57 seconds; this being the solar year by the exactest computation. The Julian year consists of 13 months 1 day and 6 hours.

## A D D I T I O N   o f   D I F F E R E N T D E N O M I N A T I O N S.

**RULE.** **B**EGIN with the column of the lowest denomination; and after adding it, you will divide the amount thereof by so many of this denomination as are equal to one of the next superior; your answer in the quot must be carried to the subsequent column, and the overplus, if any there be, must be set down underneath its own denomination; and thus proceed till you come to the last column, which add as addition of integers.

### E X A M P L E S   o f   M O N E Y.

£.	s.	d.	£.	s.	d.
7456	17	8½	89463	19	2½
5143	15	11½	24554	16	8½
6531	12	8	32222	12	10½
7476	14	7½	57086	13	6½
4144	16	9½	77418	18	8½
<hr/>			<hr/>		
30753	17	8½	280747	1	0½

To illustrate the general rule upon the first question above, by collecting my farthings, I find they amount to 7, which I divide by 4, because so many make a penny; the answer is 1, which I add to my pence, having placed the remaining 3 below farthings;

things; I then add up my pence, which amounting to 44, this I divide by 12, the number of pence in a shilling; the answer 3 I carry to my shillings, after setting the overplus 8 beneath the pence; and adding my shillings, their sum is 77, which divided by 20, the number of shillings in a pound, gives 3 for the answer, which I carry to my pounds, having set the remaining 17 below my shillings; and I then proceed with my pounds as integers; and the same method must be followed on all the tables.

*Scots Troy Weight.**English Troy Weight.*

<i>Cwt.</i>	<i>qrs.</i>	<i>lb.</i>	<i>oz.</i>	<i>dr.</i>	<i>Lb.</i>	<i>oz.</i>	<i>pwt.</i>	<i>gr.</i>
765	3	24	15	14	567	11	19	23
444	2	21	12	11	891	9	17	21
656	0	17	14	9	483	10	15	19
392	3	18	9	15	678	8	18	22
579	2	22	13	12	946	11	16	18
843	1	19	8	7	789	9	14	16
<hr/>					<hr/>			
3682	3	13	11	4	4359	3	3	23

*Apothecaries Weight.**Scots dry Measure.*

<i>Lb.</i>	<i>oz.</i>	<i>dr.</i>	<i>scr.</i>	<i>gr.</i>	<i>Ch.</i>	<i>qrs.</i>	<i>b.</i>	<i>f.pints.</i>
456	11	7	2	19	6891	3	2	3 29
789	9	5	1	17	2345	2	1	2 27
133	10	3	2	15	6789	1	3	1 25
555	8	6	1	13	2434	0	0	0 23
678	11	4	2	18	5678	2	2	3 28
913	7	2	1	16	9123	3	3	2 26
345	4	7	2	12	4567	1	1	3 24
678	9	4	1	14	8123	3	3	2 24
<hr/>					4567	0	2	3 9
4553	2	4	0	4	<hr/>			
					50522	0	3	1 29
					<hr/>			
					<i>English</i>			

*English long Measure.*

*Time.*

<i>M. f. p. y. f. i. bc.</i>	<i>Y. d. h. m. s.</i>
789 7 39 4 $\frac{1}{2}$ 2 11 2	67543 256 23 58 49
123 6 27 5 1 9 1	8456 189 19 43 56
456 4 38 3 $\frac{1}{2}$ 2 7 2	789 156 22 39 43
789 7 16 5 2 11 2	123 333 18 26 38
45 5 24 4 $\frac{1}{2}$ 1 8 1	444 284 21 44 24
67 4 33 2 $\frac{1}{2}$ 2 9 2	567 355 17 17 57
<hr/>	
273 5 22 1 $\frac{1}{2}$ 2 10 1	77926 118 3 51 27

N. B. In the above example of time, you must add up the column under days, as if it was integers, and divide the amount thereof by 365.

EXAMPLES for PRACTICE.

*Money. Troy Weight. Avoirdupoise Weight.*

<i>£. s. d. Oz. pwt. gr.</i>	<i>Lb. oz. d.</i>
172 4 7 $\frac{1}{4}$ 7 15 21	159 15 14
325 13 5 3 17 6	272 14 10
271 0 7 $\frac{1}{4}$ 2 5 14	303 15 11
524 19 1 3 16 19	255 10 4
379 14 3 $\frac{1}{4}$ 9 18 23	973 6 2
215 15 9 7 15 14	605 13 14
709 13 5 $\frac{1}{4}$ 5 18 16	517 8 9
254 17 1 $\frac{1}{2}$ 9 17 2	239 15 6
<hr/>	
<hr/>	
<hr/>	

*Cloth*

*Cloth Measure. Long Measure. Time.*

<i>Yds. qrs. n.</i>	<i>Feet. in. bc.</i>	<i>Hours m. s.</i>
35 3 2	27 9 2	52 57 35
76 2 3	35 10 1	97 48 53
95 3 0	17 2 2	35 32 45
76 1 3	35 11 1	89 16 54
25 0 1	97 8 0	25 29 18
79 2 1	82 2 1	46 44 27
54 3 2	29 3 2	75 58 59
76 0 2	14 7 1	68 27 43
<hr/>		

A man borrowed a sum of money, and part being paid of 57 l. 3 s. the remainder was 52 l. 6 s. What was the sum borrowed? *Ans.* 109 l. 9 s.

A man took a house for 12 years, and by agreement was to pay 100 l. 10 s. down, 114 l. 15 s. at the end of 6 years, and 154 l. 15 s. at the end of 12 years; how much was the whole sum? *Ans.* 370 l.

What is the estate worth *per annum*, when the taxes are 21 guineas, the precise income 8 score 19 l. 14 s.? *Ans.* 201 l. 15 s.

A shopkeeper having opened a shop, the first week sold goods to the value of fourscore pounds; the second, threescore and 5 l. the third, 43 l. 3 s. and the fourth, but 97 s. 6 d; how much did he receive in the month? *Ans.* 193 l. 6 d.

A gentleman left his daughter who was eldest 1500 l. more than the youngest, and her fortune was 11 thousand 11 hundred and 11 l. What was the eldest sister's fortune? *Ans.* The eldest sister's fortune 13611 l. and the father left them 25722 l.

S U B.

SUBTRACTION of DIFFERENT  
DENOMINATIONS.

RULE. **S**UBTRACT as in integers; only, when any of the lower denominations is greater than the upper, borrow as many of that as make one of the next superior, adding it to the upper, from which take the lesser; set down the difference, and carry one to the next lower denomination for what you borrowed.

EXAMPLES.

	£.	s.	d.	C.	qr.	lb.	oz.
I borrowed	4532	12	$7\frac{1}{4}$	562	1	22	11
I paid back	3054	15	$8\frac{1}{4}$	384	3	25	14
	<hr/>			<hr/>			
Remains unpaid	877	16	$10\frac{1}{4}$	177	1	24	13

In the above example of money, I say, 2 from 1 I cannot, but borrowing 4 farthings, which are equal to 1 penny, and adding them to 1, they make 5; and then 2 from 5, and 3 remains: and carrying 1 to 8, makes 9, which I cannot take from 7; but borrowing 12 to 7, makes 19; from which I take 9, and 10 remains; and then carrying 1 to 15, makes 16, which I cannot take from 12, but borrowing 20 to 12, makes 32, from which I take 16, and 16 remains: and thus one must proceed in every other question, according to the different tables, till you come to the last column of pounds or yards, &c. where you subtract as formerly in integers.

I borrowed of A B 729 l. 19 s. 4 d. from d<sup>o</sup>  
217 l. 16 s. 9 d. and lastly 546 l. 9 s.  $9\frac{1}{4}$  d. whereof  
† D I

# 38 SUBTRACTION of

I paid him 1378 l. 18 s. 10½ d.; how much do I still owe him?

£.	s.	d.	
729	19	4	
217	16	9	
546	9	9½	
<hr/>			
1494	5	10½	total sum borrowed.
1378	18	10½	paid.
<hr/>			
115	6	11½	remains unpaid.

Note, That, in this and similar questions, you must add the several sums borrowed or paid before you subtract.

What sum of money added to 376 l. 12 s. 3 d. will make 1000 l.?

£.	s.	d.
1000	0	0
376	12	3
<hr/>		
Ans. 623	7	9

I borrowed of my friend 749 l. 14 s. 8½ d. whereof I paid him at one time 246 l. 15 s. 9 d. and at another time 386 l. 8 s. 8 d. and lastly 99 l. 11 s. 6¾ d. I demand how much I still owe him?

£.	s.	d.		£.	s.	d.
246	15	9		Borrowed	749	14 8½
386	8	8		Paid in all	732	15 11½
99	11	6¾		<hr/>		
<hr/>			Remains unpaid	16	18	8½
732	15	11¾				

# DIFFERENT DENOMINATIONS. 39

I bought a piece of cloth containing 124 yards, whereof I sold to A B 35 yards, to C D 29 yards, to B D 44 yards 3 qrs. 2 nails; how much remains unfold?

<i>y. q. n.</i>		<i>y. q. n.</i>
A B 35 0 0	Bought	124 0 0
C D 29 0 0	Total fold	108 3 2
B D 44 3 2		<hr/>
<hr/>	Unfold	15 0 2
108 3 2		

I received from A B 245 Cwt. of coffee; whereof I sold to E D 77 Cwt. 2 qrs. to E A 54 Cwt. 3 qrs. to C D 33 Cwt. 2 qrs. and G H 69 Cwt. 3 qrs. 27 lb. 12 oz. *Quer.* how much remains unfold?

<i>Cwt. qrs. lb. oz.</i>		<i>Cwt. qrs. lb. oz.</i>
E D 77 2 0 0	Received	245 0 0 0
E A 54 3 0 0	Sold	235 2 27 12
C D 33 2 0 0		<hr/>
G H 69 3 27 12	Unfold	9 1 0 4
<hr/>		
235 2 27 12		

A brewer, from a hoghead of ale, sent one of his customers 6 gallons, and used in his own family 4 gallons 2 quarts 1 pint. *Quer.* how much remained in said hoghead?

<i>G. q. p.</i>		<i>Hog. g. q. p.</i>
6 0 0		1 0 0 0
4 2 1		0 10 2 1 drawn off.
<hr/>		<hr/>
10 2 1		0 5 1 1 remain.
	D 2	How

# 40 SUBTRACTION, &c.

How old is the man that was born in the year 1664 on the 15th of June at 11 o'clock, this being the 22d of March 1770 at 6 o'clock in the morning?

$$\begin{array}{r}
 1769 \quad 2 \quad 21 \quad 6 \\
 1663 \quad 5 \quad 14 \quad 11 \\
 \hline
 \text{Years } 105 \quad 9 \quad 6 \quad 19
 \end{array}$$

A gentleman had an estate of 500 acres, whereof he set out to each of 5 tenants, 88 acres 2 roods 18 falls. *Quaritur*, how much remained in his own hand?

<i>Acres</i>	<i>r.</i>	<i>f.</i>	<i>A.</i>	<i>r.</i>	<i>f.</i>	
88	2	18	500	0	0	estate.
		5	443	0	10	set off.
<hr/>			<hr/>			
413	0	10	56	3	30	remained.

A is indebted to the brewer the sum of 117 l. 2 s. 5 d. and B owes him 273 l. how much does the one owe more than the other? *Ans.* 155 l. 17 s. 7 d.

When an estate of 300 l. *per annum* is reduced, on paying of taxes, to 12 score and 14 l. 6 s. what is the tax? *Ans.* 45 l. 14 s.

A horse in his furniture is worth 37 l. 5 s. out of it he is worth only 14 guineas: how much does the price of the furniture exceed that of the horse? *Ans.* 7 l. 17 s.

A merchant, at his outsetting in trade, owed 750 l. he had in cash, commodities, the stocks, good debts, 12510 l. 7 s. He cleared the first year by commerce, 452 l. 3 s. 5 d. What was his real

# MULTIPLICATION, &c. 41

real balance at the 12 months end? *Ans.* 12212 l.  
10 s. 5 d.

## MULTIPLICATION of DIFFERENT DENOMINATIONS.

**RULE 1.** IF the quantity be any one of the nine digits, place it under the lowest denomination, multiply it into all the parts, and carry according to the next denomination, as in the following

### EXAMPLE.

What cost 9 Cwt. of sugar at 2 l. 8 s.  $6\frac{1}{2}$  d.  
per Cwt.?

£.	s.	d.
2	8	$6\frac{1}{2}$
		9
<hr/>		
£ 21	16	$10\frac{1}{2}$

**RULE 2.** If your quantity is a composite number, resolve it into its component parts, which must be all digits, or 12; then multiply the sum given by any one of these parts, and the product arising therefrom by the other; and thus proceed till you have exhausted all your component parts.

D 3

E X.

# 42 MULTIPLICATION of

## EXAMPLES.

What cost 72 hogheads of strong ale, at 2 l. 8 s. 8½ d. per hoghead?

$$8 \times 9 = 72$$

£.	s.	d.
2	8	8½
<hr/>		
19	9	8
<hr/>		
		9
<hr/>		
£. 175	7	0

What cost 81 lb. of tea, at 9 s. 4½ d. per lb.?

$$9 \times 9 = 81$$

£.	s.	d.
	9	4½
<hr/>		
4	4	2½
<hr/>		
		9
<hr/>		
£ 37	17	8½

What cost 144 yards of velvet, at 1 l. 4 s. 10½ d. per yard?

$6 \times 6 \times 4 = 144$ $12 \times 12 = 144$	<table> <thead> <tr><th>£.</th><th>s.</th><th>d.</th></tr> </thead> <tbody> <tr><td>1</td><td>4</td><td>10½</td></tr> <tr><td colspan="3"><hr/></td></tr> <tr><td></td><td></td><td>12</td></tr> <tr><td colspan="3"><hr/></td></tr> <tr><td>14</td><td>18</td><td>6</td></tr> <tr><td colspan="3"><hr/></td></tr> <tr><td></td><td></td><td>12</td></tr> <tr><td colspan="3"><hr/></td></tr> <tr><td>£ 179</td><td>2</td><td>0</td></tr> </tbody> </table>	£.	s.	d.	1	4	10½	<hr/>					12	<hr/>			14	18	6	<hr/>					12	<hr/>			£ 179	2	0	<table> <thead> <tr><th>£.</th><th>s.</th><th>d.</th></tr> </thead> <tbody> <tr><td>1</td><td>4</td><td>10½</td></tr> <tr><td colspan="3"><hr/></td></tr> <tr><td></td><td></td><td>6</td></tr> <tr><td colspan="3"><hr/></td></tr> <tr><td>7</td><td>9</td><td>3</td></tr> <tr><td colspan="3"><hr/></td></tr> <tr><td></td><td></td><td>6</td></tr> <tr><td colspan="3"><hr/></td></tr> <tr><td>44</td><td>15</td><td>6</td></tr> <tr><td colspan="3"><hr/></td></tr> <tr><td></td><td></td><td>4</td></tr> <tr><td colspan="3"><hr/></td></tr> <tr><td>£ 179</td><td>2</td><td>0</td></tr> </tbody> </table>	£.	s.	d.	1	4	10½	<hr/>					6	<hr/>			7	9	3	<hr/>					6	<hr/>			44	15	6	<hr/>					4	<hr/>			£ 179	2	0
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N. B.

# DIFFERENT DENOMINATIONS. 43

N. B. If your multiplier is not a composite number, whose component parts are digits, first multiply the sum given by a composite number less than your multiplier, but let it be as near to it as possible; then multiply the given sum by the difference of the multiplier and that nearest composite number; which two products, when added, give the answer.

What cost 79 bolls of wheat, at 18 s. 10½ d. per boll?

$$8 \times 9 + 7 = 79$$

£.	s.	d.	£.	s.	d.
0	18	10½	0	18	10½
		8			7
<hr/>			<hr/>		
7	11	2	6	12	3½
		9			
<hr/>			<hr/>		
68	0	6			
6	12	3½			
<hr/>			<hr/>		
£ 74	12	9½			

What cost 104 bags of flour, at 1 l. 14 s. 6½ d. per bag?

# 44 MULTIPLICATION of

$$12 \times 8 + 8 = 104$$

£.	s.	d.	£.	s.	d.
1	14	$6\frac{1}{2}$	1	14	$6\frac{1}{2}$
		12			8
<hr/>			<hr/>		
20	14	9	£ 13	16	6
		8			
<hr/>			<hr/>		
165	18	0			
13	16	6			
<hr/>			<hr/>		
£ 179	14	6			

What is the weight of 52 boxes of goods, each weighing 2 Cwt. 3 qrs. 17 lb.?

$$52 = 6 \times 8 + 4$$

Cwt.	qrs.	lb.	Cwt.	qrs.	lb.
2	3	17	2	3	17
		6			4
<hr/>			<hr/>		
17	1	18	11	2	12
		8			
<hr/>			<hr/>		
139	1	4			
11	2	12			
<hr/>			<hr/>		
Cwt. 150	3	16			

What

# DIFFERENT DENOMINATIONS. 45

What cost 1008 stones of hay, at  $4\frac{1}{2}$  d. per stone?

$$8 \times 5 \times 5 \times 5 + 8 = 1008 \quad \begin{array}{r} d. \\ 4\frac{1}{2} \\ 8 \end{array}$$


---


$$\begin{array}{r} 3 \quad 0 \\ 5 \\ \hline 15 \quad 0 \\ 5 \\ \hline 3 \quad 15 \quad 0 \\ 5 \\ \hline 18 \quad 15 \quad 0 \\ 3 \quad 0 = \text{the price of 8 stones hay.} \\ \hline 18 \quad 18 \quad 0 \end{array}$$

What cost 99 lb. of tea, at 9 s.  $8\frac{1}{2}$  d. per lb.?

$$12 \times 8 + 3 = 99 \quad \begin{array}{r} \text{£.} \quad \text{s.} \quad \text{d.} \\ 0 \quad 9 \quad 8\frac{1}{2} \\ 12 \\ \hline 5 \quad 16 \quad 6 \\ 8 \quad \text{£.} \quad 1 \quad 9 \quad 1\frac{1}{2} \\ \hline 46 \quad 12 \quad 6 \\ 1 \quad 9 \quad 1\frac{1}{2} = \text{the price of 3 lb. tea.} \\ \hline \text{£.} \quad 48 \quad 1 \quad 1\frac{1}{2} \end{array}$$

E X.

## 46 MULTIPLICATION of

### EXAMPLES for PRACTICE.

75 lb. of nutmegs, at 7 s.  $2\frac{1}{4}$  d. per lb. *Facit*  
27 l. 2 s.  $2\frac{3}{4}$  d.

93 Cwt. of cheese, at 1 l. 5 s. 3 d. per Cwt. *Facit*  
117 l. 8 s. 3 d.

127 lb. of Bohea tea, at 12 s. 3 d. per lb. *Facit*  
77 l. 15 s. 9 d.

135 gallons of rum, at 7 s. 5 d. per gal. *Facit*  
50 l. 1 s. 3 d.

6 doz. pair of gloves at 1 s. 10 d. per pair. *Facit*  
6 l. 12 s.

$7\frac{1}{4}$  Cwt. of raisins, at 1 l. 1 s. 6 d. per Cwt. *Facit*  
7 l. 15 s.  $10\frac{1}{2}$  d.

$6\frac{1}{2}$  barrels of herrings, at 3 l. 15 s. 7 d. *Facit*  
24 l. 11 s.  $3\frac{1}{2}$  d.

$56\frac{1}{4}$  Cwt. of sugar, at 2 l. 18 s. 7 d. per Cwt. *Facit*  
166 l. 4 s.  $7\frac{1}{4}$  d.

$87\frac{1}{4}$  bushels of wheat, at 4 s. 3 d. per bushel.  
*Facit* 18 l. 12 s.  $11\frac{1}{4}$  d.

$29\frac{1}{4}$  lb. of fine tea, at 1 l. 3 s. 6 d. per lb. *Facit*  
34 l. 7 s.  $4\frac{1}{2}$  d.

$96\frac{1}{2}$  Cwt. of currants, at 2 l. 15 s. 6 d. per Cwt.  
*Facit* 267 l. 15 s. 9 d.

There

## DIFFERENT DENOMINATIONS. 47

There are 124 men employed to finish a piece of work, and they are to have 5 l. each man; how much will they have in all? *Ans.* 620 l.

There were 25 men concerned in the payment of a sum of money, and each man paid 5 guineas; how much was paid in all? *Ans.* 131 l. 5 s.

The sum of two numbers is 360, the less of them 144; what is their product, and square of their difference? *Ans.* 31104 product, 5184 square of their difference.

What did that gentleman receive in dowery with his wife, whose fortune was her wedding-suit; her petticoat having 2 rows of furbelows, each furbelow 87 quills, and each quill 21 guineas? *Ans.* 3836 l. 14 s.

A merchant had 19118 l. to begin trade with. For 5 years together he cleared 1086 l. a-year, the next 4 years he made good 2715 l. 10 s. 6 d. a-year; but the last 3 years he was in trade, had the misfortune to lose, one year with another, 475 l. 4 s. 6 d. a-year: what was his real fortune at 12 years end? *Ans.* 33984 l. 8 s. 6 d.

I shall finish this rule by observing, how absurd it is to propose (as some authors have done) to multiply, for example, 4 l. 6 s. 8 d. by 3 l. 2 s. 6 d. &c. If the proposers of such questions would be so good as tell us, how oft they would have such a sum taken or repeated, (which must be the meaning of multiplication, else it has no meaning at all), I should do my best to give them a satisfactory answer; but till they explain themselves, I think they deserve none. My intended brevity does not allow me to prosecute this subject, and therefore I must refer the reader to Malcolm's arithmetic, page 85. London edition, 1730.

# DIVISION of DIFFERENT DENOMINATIONS.

Divide 456 l. among 36 men ?

	<i>£.</i>	<i>s.</i>	<i>d.</i>
36)456	12	13	4
36			
<hr/>			
	96		
	72		
	<hr/>		
	24		
	20		
	<hr/>		
	480		
	36		
	<hr/>		
	120		
	108		
	<hr/>		
	12		
	12		
	<hr/>		
	144		
	144		
	<hr/>		

Divide

# DIFFERENT DENOMINATIONS. 49

Divide 1568 l. 19 s. 4 d. among 88 men.

$$88)1568 : 19 : 4(17 : 16 : 7$$

88

---

688

616

---

72

20

---

1459

88

---

579

528

---

51

12

---

616

616

---

N. B. In these and similar questions, after finding the integers contained in the quotient, when you multiply the remainder by 20, you must add your shillings, and then ask how oft your divisor is contained in this new dividend, and place the answer in your quotient; and if any shillings then remain, multiply them by 12, and add thereto your pence; then divide as before; and if any pence remain, multiply the same by 4, and adding your farthings, continue your division as formerly. The same method must be pursued with questions that fall under different tables; as for instance, divide 76548 acres among 245 men.

† E

249

## DIVISION of

245)76548(312 : 1 : 3 : 19 $\frac{3}{4}$ 

735

304

245

598

490

108

4

432

245

187

40

7480

735

130

36

780

390

4680

245

2230

2205

(25)

A captain and 160 men gain a prize of 368 l.  
whereof the captain was to have  $\frac{1}{7}$ ; and the re-  
mainder

# DIFFERENT DENOMINATIONS. 51

mainder was to be divided equally among the men. *Quer.* each man's share. *Ans.* The captain got 73 l. 12 s. and each of the men 1 l. 16 s. 9½ d.

5)368

73 : 12=captain's share.

160)294 : 8(1 : 16 : 9½=each man's share.

160

134

20

2688

160

1088

960

128

12

1536

1440

96

4

384

320

64

If a man spends 257 l. 2 s. 5 d. in 12 months time, what is that per month? *Ans.* 21 l. 8 s. 6½ d.

The cloathing of 35 charity-boys came to 57 l.

E 2

3 s.

3 s. 7 d. what is the expence of each ? *Ans.* 1 l.  
12 s. 8 d.

If 20 Cwt of tobacco came to 27 l. 5 s.  $4\frac{1}{2}$  d.  
at what rate is that per Cwt. ? *Ans.* 1 l. 7 s. 3 d.

What is the value of one hogshead of beer,  
when 120 are sold at 154 l. 17 s. 10 d. ? *Ans.* 1 l.  
5 s.  $9\frac{1}{4}$  d.

A prize of 7257 l. 3 s. 6 d. is to be equally divi-  
ded amongst 500 sailors, what is each man share ?  
*Ans.* 14 l. 10 s.  $3\frac{1}{4}$  d.

What number is that which multiplied by 7847,  
will make the product 3013248 ? *Ans.* 384.

## REDUCTION.

**R**EDUCTION expresses the same value in differ-  
ent numbers, under different names ; for  
we say that 4 pounds are of the same value with 80  
shillings or 960 pence.

Reduction is threefold, descending, ascending,  
or mixt.

Reduction descending brings a greater name to  
a lesser, as pounds to shillings. **RULE :** Multiply  
by the value of the greater name.

Reduction ascending brings a lesser name to a  
greater, as hours to days. **RULE :** Divide by the  
value of the greater name.

Mixt Reduction brings a greater name to a lesser,  
or a lesser to a greater. **RULE :** Find a third name  
that is contained in the name given, and in the  
name sought, a just number of times ; reduce the  
name given to that third name, by rule 1. then  
reduce that third name to the name sought, by  
rule 2.

E X.

# REDUCTION DESCENDING. 53

## EXAMPLES OF REDUCTION DESCENDING.

REDUCE 6783 l. to farthings

$$\begin{array}{r}
 6783 \\
 \times 20 \\
 \hline
 135660 \\
 \times 12 \\
 \hline
 1627920 \\
 \times 4 \\
 \hline
 6511680 \text{ farthings.}
 \end{array}$$

N. B. If there are shillings, pence, and farthings, you must remember to add these in their respective places, as in the next example.

Reduce 456 l. 13 s. 4 d, and 2 far. to shillings, pence, and farthings.

$$\begin{array}{r}
 456 : 13 : 4\frac{1}{2} \\
 \times 20 \\
 \hline
 9133 \text{ shillings.} \\
 \times 12 \\
 \hline
 109600 \text{ pence.} \\
 \times 4 \\
 \hline
 438403 \text{ farthings.}
 \end{array}$$

E 3

Reduce

# 54 REDUCTION DESCENDING.

Reduce 469 yards to nails.

$$\begin{array}{r}
 469 \\
 4 \\
 \hline
 1876 \\
 4 \\
 \hline
 7504 \text{ nails.}
 \end{array}$$

Reduce 468 Cwt. to quarters, pounds, and ounces.

$$\begin{array}{r}
 468 \\
 4 \\
 \hline
 1872 \text{ quarters.} \\
 28 \\
 \hline
 14976 \\
 3744 \\
 \hline
 52416 \text{ pounds,} \\
 16 \\
 \hline
 314496 \\
 52416 \\
 \hline
 838656 \text{ ounces.}
 \end{array}$$

In 12 l. how many shillings, pence, and farthings? *Ans.* 240 s. 2880 d. 11520 far.

How many farthings are there in 21 guineas? *Ans.* 21168.

In 27 ounces of gold how many grains? *Ans.* 12960.

In 3 lb. 10 oz. 7 pwt. 5 gr. how many grains? *Ans.* 22253.

# REDUCTION ASCENDING. 55

## REDUCTION ASCENDING.

### EXAMPLES.

In 11520 farthings, how many pence, shillings, and pounds?

$$\begin{array}{r}
 4) 11520 \\
 \hline
 12) 2880 \text{ pence.} \\
 \hline
 20) 240 \text{ shillings.} \\
 \hline
 12 \text{ pounds.}
 \end{array}$$

In 968452 farthings, how many guineas?

	12	21	guin. s. d.
4) 968452	(242113	(20176	(960:16:1
8	24	189	or thus 4) 968452
—	—	—	—
16	21	127	12) 242113
16	12	126	—
—	—	—	21=7×3 7) 20176:1
8	91	16	—
8	84		3) 28824:2
—	—		—
5	73		960:16:1
4	72		
—	—		
12	(1)		
12			
—			

## 56 MIXT REDUCTION.

In 7684 nails, how many yards?

$$\begin{array}{r} 4 \overline{) 7684} \\ \hline \end{array}$$

$$\begin{array}{r} 4 \overline{) 1921} \\ \hline \end{array}$$

480 yards 1 qr.

In 42161 grains, how many pounds Troy?

$$24 = 6 \times 4 \quad 4 \overline{) 42161}$$

$$\begin{array}{r} 6 \overline{) 10540 : 1} \\ \hline \end{array}$$

$$\begin{array}{r} 20 \overline{) 1756 : 17} \\ \hline \end{array}$$

$$\begin{array}{r} 12 \overline{) 87 : 16} \\ \hline \end{array}$$

lb. Troy 73 : 16 : 17

How many pence, shillings, and pounds are there in 17280 farthings. *Ans.* 4320 d. 360 s. 18 l.

In 900 pence, how many shillings and crowns?  
*Ans.* 75 s. 15 crowns.

## MIXT REDUCTION.

### EXAMPLES.

In 25796 merks Scots, how many pounds Scots?

$$\begin{array}{r} 25796 \\ 2 \\ \hline \end{array}$$

2

$$\begin{array}{r} 3 \overline{) 51592} \text{ half-merks.} \\ \hline \end{array}$$

17197 pounds.

In

# MIXT REDUCTION. : 57

In 75648 merks Scots, how many pounds Sterling?

$$\begin{array}{r}
 1 \text{ l. Ster.} = 18 \text{ m. \& } 18 = 6 \times 3 \quad 6)75648 \\
 \hline
 \quad \quad \quad 3)12608 \\
 \hline
 \quad \quad \quad \pounds. 4202\frac{2}{3} \text{ or } 17 \text{ s. } 4 \text{ d.}
 \end{array}$$

In 976800 pieces of eight, at 4 s. 6 d. each, how many pounds Sterling?

$$\begin{array}{r}
 976800 \\
 \quad \quad 9 \\
 \hline
 40)8791200 \\
 \hline
 \pounds. 219780
 \end{array}$$

In

# 58 MIXT REDUCTION.

In 796 l. Sterling, how many French livres, at  $15\frac{1}{2}$  d. each?

$$\begin{array}{r}
 796 \\
 20 \\
 \hline
 15920 \\
 24 \\
 \hline
 63680 \\
 31840 \\
 \hline
 \text{Livres. sols. deniers.} \\
 31) 382080 (12325 : 3 : 2\frac{3}{4} \\
 31 \\
 \hline
 72 \\
 62 \\
 \hline
 100 \\
 93 \\
 \hline
 78 \\
 62 \\
 \hline
 160 \\
 155 \\
 \hline
 5 \\
 20 \\
 \hline
 100(3 \\
 93 \\
 \hline
 7 \\
 12 \\
 \hline
 84(2 \\
 62 \\
 \hline
 (22)
 \end{array}$$

# MIXT REDUCTION. 59

A butcher sent his servant to a fair with 360 l. and ordered him to buy bullocks at 5 l. 5 s. each, cows at 3 l. 12 s. weathers at 7 s. 6 d. and lambs at 3 s. 8 d. and of each an equal number; the servant was allowed 3 s. 9 d. of expences. *Quer.* how many of each kind he brought home, and how much cash he had to return to his master?

360	l.	s.	d.
20	5	5	0
<hr/>	3	12	0
7200	0	7	6
12	0	3	8
<hr/>	<hr/>	<hr/>	<hr/>
86400 pence received.	9	8	2
	20		
	<hr/>		
	188		
	12		
	<hr/>		
	2258		Pence.

2258)86400(38 of each

6774

---

18660

18064

---

596

45=cash allowed the servant.

---

551=2 l. 5 s. 11 d. returned the master.

In 468 l. Sterling, how many crowns, half-crowns, shillings, sixpences, and threepences, and of each an equal number?

*Ans.* 1011.

How

How many merks at 13 s. 4 d. and pistoles at 17 s. 6 d. are contained in 314 l. 12 s. 6 d.? *Ans.* 471 merks and 359 pistoles.

In 29530 French crowns, at 4 s. 6 d. how many pounds Sterling? *Ans.* 6644 l. 5 s.

In 76543 bitts of Jamaica, at  $7\frac{1}{2}$  d. each, how many pounds Sterling? *Ans.* 2391 l. 19 s.  $4\frac{1}{2}$  d.

In 150000 crusades, each 400 reis, 1000 for a milrea at 5 s. 6 d. how many pounds Sterling? *Ans.* 16500 l.

In 478 Cwt. of lead, how many foddors of d<sup>o</sup> at  $19\frac{1}{2}$  d. Cwt. each? *Ans.* 24 foddors.

In 17 piggs of lead at 3 Cwt. 3 qrs. each, how many foddors? *Ans.* 3 foddors.

How many spoons, each 2 oz. 12 pwt. 14 gr. may be made out of 250 oz. of silver? *Ans.* 95 spoons.

In 79640 ounces of silver, how many snuff-boxes may be made, each weighing  $3\frac{3}{4}$  oz.? *Ans.* 21237 boxes.

How many barley-corns will surround the globe of the earth, whose circumference is 360 degrees? *Ans.* 4763266560 barley-corns.

How many seconds are there since the birth of our Saviour, this being December 31. 1769? *Ans.* 55824221553 seconds.

How many parcels of sugar, of 16 lb. 2 oz. are there in 16 Cwt. 1 qr. 15 lb.? *Ans.* 113 parcels 12 lb. 14 oz. Avoirdupoise weight.

The allowances commonly made in this weight are, tare, trett, and cloff, which we shall treat of briefly in this place.

Tare is allowance made to the buyer for the weight of the box, barrel, or bag, &c. which contains the goods bought; and is either

At so much per box, barrel, bag, &c.

At so much per cent. or

At so much in gross weight.

Trett

# MIXT REDUCTION. 61

Trett is an allowance of 4 lb. in every 104 lb. for waste, dust, &c. made by the merchant to the buyer.

Cloff is an allowance of 2 lb. to the citizens of London on every draught above 3 Cwt. on some sort of goods.

Gross weight is the whole weight of any sort of goods, and that which contains them.

Suttle is when part of the allowance is deducted from the gross.

Neat is the pure weight, when all allowances are deducted.

**RULE 1.** When the tare is at so much per bag, barrel, &c. multiply the number of bags, barrels, &c. by the tare, and subtract the product from the gross; the remainder is neat.

## EXAMPLES.

In 7 fraills of raisins, each weighing 5 Cwt. 2 qrs. 5 lb. gross, tare at 23 lb. per fraill, how much neat weight?

$$\begin{array}{r}
 23 \\
 7 \\
 \hline
 28 \overline{)161} \quad (5 \text{ } 1 \\
 140 \quad 4 \\
 \hline
 21 \quad 1
 \end{array}$$

$$\begin{array}{r}
 \text{Cwt. qrs. lb.} \\
 5 \quad 2 \quad 5 \\
 \quad \quad 7 \\
 \hline
 38 \quad 3 \quad 7 \text{ gross weight.} \\
 1 \quad 1 \quad 21 \text{ tare.} \\
 \hline
 37 \quad 1 \quad 14 \text{ neat.}
 \end{array}$$

What is the neat weight of 25 hogshheads of tobacco, weighing gross 163 Cwt. 2 qrs. 15 lb. tare 100 lb. per hogshhead? *Ans.* 141 Cwt. 1 qr. 7 lb.

† F

RULE

# 62 MIXT REDUCTION.

**RULE 2.** When the tare is at so much in the whole gross weight, subtract the given tare from the gross; the remainder is neat.

In 3 hogsheds of tobacco, containing as under, how much neat weight?

	Cwt. qr. lb.			
N <sup>o</sup> 1.	5	1	2	tare 105 lb.
2.	3	2	17	83
3.	4	1	15	92

*Ans.* 10 Cwt. 3 qrs. 6 lb.

**RULE 3.** When the tare is so much per cent. divide the gross weight by the aliquot parts of a Cwt. which subtract from the gross; the remainder is neat.

Note 7 lb. =  $\frac{1}{8}$  and 14 lb. =  $\frac{1}{4}$   
 8 =  $\frac{1}{4}$  16 =  $\frac{1}{2}$

What is the neat weight of 18 butts of currants, each 8 Cwt. 2 qrs. 5 lb. tare at 14 per cent.?

	Cwt. qr. lb.		
18 = 9 × 2	8	2	5
			9
	76	3	17
			2
14 = $\frac{1}{4}$	153	3	6
	19	0	25 $\frac{1}{4}$
	Cwt. 134	2	8 $\frac{1}{4}$

**RULE**

# MIXT REDUCTION. 63

**RULE 4.** When trett is allowed with tare, divide the pounds futtle by 26, the quotient is the trett; which subtract from the futtle, the remainder is neat.

In 37 butts of currants, each 12 Cwt. 2 qrs. 24 lb. gross, tare 14 lb. per cent. trett 4 per 104 lb. how many pounds neat?

$$\begin{array}{r}
 \text{Cwt. qr. lb.} \\
 12 \quad 2 \quad 24 \\
 4 \\
 \hline
 50 \\
 28 \\
 \hline
 14 = \frac{5}{8} \quad 1424 \text{ gross.} \\
 \quad \quad 178 \text{ tare.} \\
 \hline
 26) 1246 \text{ futtle.} \\
 \quad 47 \text{ trett.} \\
 \hline
 1199 \text{ neat.}
 \end{array}$$

In 7 Cwt. 3 qrs. 27 lb. gross, tare 36 lb. trett 4 lb. per 104 lb. how many pounds neat? *Ans.* 826 lb.

**RULE 5.** When cloff is allowed, divide the Cwts. (after trett is taken) by 3, the quotient is so many double lbs. which multiply by 2 to bring them into pounds, or divide by 56 to bring them into Cwts. subtract it from the futtle, the remainder is neat.

## 64 MIXT REDUCTION.

What is the neat weight of three hogsheds of tobacco, weighing 15 Cwt. 3 qrs. 20 lb. gross, tare 7 lb. per cent. trett 4 lb. per 104 lb. cloff 2 lb. for 3 Cwt.?

	<i>Cwt. qr. lb.</i>
<i>Ans.</i> 14 Cwt. 1 qr. 3 lb.	$7\frac{1}{8})15 \quad 3 \quad 20 \text{ gross.}$
	$3 \quad 27\frac{1}{2} \text{ tare.}$
	<hr/>
	$1\frac{1}{8})14 \quad 3 \quad 20\frac{1}{2} \text{ futtle.}$
	$2 \quad 8 \text{ trett.}$
	<hr/>
	$14 \quad 1 \quad 12\frac{1}{2} \text{ futtle.}$
	$9\frac{1}{2} \text{ cloff}$
	<hr/>
	$14 \quad 1 \quad 3 \text{ neat.}$

In 7 hogsheds of tobacco, each weighing gross 5 Cwt. 2 qrs. 7 lb. tare 8 lb. per cent. trett 4 lb. per 104 lb. cloff 2 lb. per 3 Cwt. how much neat weight? *Ans.* 34 Cwt. 2 qrs.  $7\frac{3}{4}$  lb.

A factor has sold goods at Cadiz for 1468 pieces of eight, at 4 s.  $6\frac{1}{2}$  d. per piece; how much Sterling is the sum? *Ans.* 333 l. 7 s. 2 d.

If a bill is drawn from Lisbon at 1432 mill-reas, at 6 s. 8 d. per piece; how much English money is that bill? *Ans.* 477 l. 6 s. 8 d.

A bill of 220 l. 16 s. 8 d. is drawn from London; what is the value at Florence in ducatoons, at 53 d. each? *Ans.* 1000 ducatoons.

If 100 florins, at  $59\frac{1}{2}$  d. each, be remitted from Francfort to London; what is the value in pounds Sterling? *Ans.* 24 l. 15 s. 10 d.

There are 800 French crowns, at 4 s. 6. each, remitted to London by a merchant from Paris; what

what is the value in pounds Sterling? *Ans.* 180 l. Sterling.

## REDUCTION of VULGAR FRACTIONS.

**I**N imitation of former authors, (though but few in number), we shall here treat of reduction of Vulgar Fractions and Decimals, prior to the rule of Proportion; for this obvious reason, because many questions will occur in said rule, which have a remainder; and that remainder is always a vulgar fraction, which, by annexing ciphers, can easily be reduced to a decimal; which will render the operation much shorter, and by a little practice will be equally perspicuous: however, great regard must be had to that rule which follows, relating to reduction of decimals by inspection.

**DEFINITION 1.** A fraction is a part or parts of unity or any whole; as  $\frac{1}{2}$  or  $\frac{3}{4}$ .

**DEF. 2.** A proper fraction is less than its whole, or whose numerator is less than its denominator; as  $\frac{4}{7}$  or  $\frac{8}{15}$ .

**N. B.** The denominator is so called, because it denominates, or gives name to the several fractions arising from the division of the whole, and is placed below a line; whereas the number above the line shews how many of these parts the fraction contains, and is therefore called the numerator; so the numerator is the fraction, and the denominator only the name of it.

**F 3:**

**DEF.**

DEF. 3. An improper fraction is equal to, or greater than its whole, or whose numerator is equal to, or greater than its denominator; as  $\frac{4}{3}$  or  $\frac{7}{3}$ .

DEF. 4. A simple fraction has but one numerator and one denominator; as  $\frac{2}{3}$  or  $\frac{7}{6}$ .

DEF. 5. A compound fraction joins two or more simple fractions together, with the particle *of* betwixt them, and is a fraction of a fraction; as  $\frac{2}{3}$  of  $\frac{4}{5}$ .

DEF. 6. A mixt number is composed of an integer and a fraction: as  $8\frac{3}{4}$ .

RULE 1. To reduce improper fractions to integers or mixt numbers: Divide the numerator by the denominator, the quot gives integers; the remainder is the numerator of the fraction to be annexed, whose denominator is the same as before.

### EXAMPLES.

Reduce  $77\frac{6}{8}$  to an integer.

$$\begin{array}{r} 8 \overline{) 776} \\ \underline{\phantom{0}00} \text{ Ans. } 97 \\ 97 \end{array}$$

Reduce  $288\frac{23}{23}$  to a mixt number.

$$\begin{array}{r} 23 \overline{) 288} (12\frac{2}{23} \\ \underline{23} \\ 58 \\ \underline{46} \\ 12 \end{array}$$

RULE

RULE 2. To reduce mixt numbers to improper fractions ; multiply the integer by the denominator ; to the product add the numerator ; that sum gives the numerator of the improper fraction, whose denominator is the same as before.

## E X A M P L E S.

Reduce  $12\frac{1}{2}$  to an improper fraction.

$$\begin{array}{r}
 12 \\
 23 \\
 \hline
 276 \\
 +12 \\
 \hline
 288 \\
 \hline
 (23)
 \end{array}
 \quad \text{Ans. } \frac{288}{23}$$

Reduce  $48\frac{1}{2}$  to an improper fraction.

$$\begin{array}{r}
 48 \\
 25 \\
 \hline
 240 \\
 96 \\
 \hline
 1200 \\
 +18 \\
 \hline
 1218 \\
 \hline
 (25)
 \end{array}
 \quad \text{Ans. } \frac{1218}{25}$$

Reduce

Reduce  $96\frac{2}{3}$  to an improper fraction.

$$\begin{array}{r}
 96 \\
 35 \\
 \hline
 480 \\
 288 \\
 \hline
 3360 \\
 +23 \\
 \hline
 3383 \\
 \hline
 (35)
 \end{array}
 \quad \text{Ans. } 338\frac{3}{35}$$

**RULE 3.** To reduce integers to fractions of a given denominator :

Multiply the integer by the given fraction for the numerator of the fraction.

### EXAMPLES.

Reduce 36 to a fraction whose denominator is 4.

$$\begin{array}{r}
 36 \\
 4 \\
 \hline
 144 \\
 \hline
 (4)
 \end{array}
 \quad \text{Ans. } 14\frac{4}{4}$$

**N. B.** To reduce an integer to the form of a fraction, is to make unity the denominator. Thus  $8 = \frac{8}{1}$  and  $12 = \frac{12}{1}$ .

P R O-

## P R O B L E M.

To find a fraction equal to a given fraction, and having a given denominator :

RULE. Multiply the numerator by the given denominator, and divide the product by the denominator of the fraction ; the quot gives the numerator of the new fraction.

## E X A M P L E S.

To find a fraction equal to  $\frac{1}{3}$  whose denominator is 36.

$$\begin{array}{r}
 4 : 3 :: 36 \\
 \quad \quad 3 \\
 \hline
 4)108(27 \quad \text{Ans. } \frac{27}{36} = \frac{1}{3} \\
 \quad \quad 8 \\
 \hline
 \quad \quad 28 \\
 \quad \quad 28 \\
 \hline
 \end{array}$$

To find a fraction equal to  $\frac{6}{9}$  whose denominator is 12.

$$\begin{array}{r}
 9 : 6 :: 12 \\
 \quad \quad 6 \\
 \hline
 9)72 \\
 \quad \quad 8
 \end{array}
 \quad \text{Ans. } \frac{8}{12} = \frac{6}{9}$$

RULE 4. To reduce compound fractions to simple fractions : Multiply the numerators continually

## REDUCTION of

tinually for the numerator of the simple fraction,  
and the denominators for its denominator.

## EXAMPLES.

Reduce  $\frac{2}{4}$  of  $\frac{5}{3}$  to a simple fraction.

$$\begin{array}{r} 2 \\ 4 \\ \hline 8 \end{array} \quad \begin{array}{r} 5 \\ 3 \\ \hline 15 \end{array} \quad \text{Ans. } \frac{2}{4} \text{ of } \frac{5}{3} = \frac{5}{12}$$

Reduce  $\frac{7}{4}$  of  $\frac{5}{3}$  of  $\frac{7}{9}$  to a simple fraction.

$$\begin{array}{r} 7 \\ 4 \\ \hline 28 \\ 2 \\ \hline 56 \end{array} \quad \begin{array}{r} 9 \\ 5 \\ \hline 45 \\ 3 \\ \hline 135 \end{array} \quad \text{Ans. } \frac{7}{4} \text{ of } \frac{5}{3} \text{ of } \frac{7}{9} = \frac{35}{144}$$

**RULE 5.** To value fractions in the known parts of the integer; multiply the numerator by the number of known parts contained in the integer, and divide the product by the denominator, the quot gives the answer.

## EXAMPLES.

What is the value of  $\frac{3}{4}$  of a pound?

$$\begin{array}{r} 3 \\ 20 \\ \hline 4)60 \\ \hline 15 \end{array} \quad \text{Ans. } \frac{3}{4} \text{ of a l.} = 15 \text{ s.}$$

What

# VULGAR FRACTIONS.

71

What is the value of  $\frac{9}{16}$  of a pound?

$$\begin{array}{r}
 9 \\
 20 \\
 \hline
 16) 180 (11 : 3 \\
 16 \\
 \hline
 20 \\
 16 \\
 \hline
 4 \\
 12 \\
 \hline
 48 (3 \\
 48 \\
 \hline
 \end{array}$$

*Ans.* 11 s. 3 d. =  $\frac{9}{16}$  of a l.

What is the value of  $\frac{8}{14}$  of a year?

$$\begin{array}{r}
 8 \\
 13 \\
 \hline
 14) 104 (7 : 1 : 5 \\
 98 \\
 \hline
 6 \\
 4 \\
 \hline
 24 (1 \\
 14 \\
 \hline
 10 \\
 7 \\
 \hline
 70 (5 \\
 70 \\
 \hline
 \end{array}$$

*Ans.*  $\frac{8}{14} = 7$   $\frac{1}{14}$   $\frac{5}{14}$

*ys. m. w. d.*

L E M.

## L E M M A.

To find the greatest common divisor to two given numbers; divide the greater by the lesser number, and the divisor by the remainder, and so on continually till unity or 0 remain. If 1 remains, the numbers have no common divisor. If 0 remains, the last divisor is the greatest common divisor.

## E X A M P L E S.

Required the greatest common divisor to 784 and 952. *Ans.* 56

$$\begin{array}{r}
 784 \overline{)952} (1 \\
 \underline{784} \\
 168(784)4 \\
 \underline{672} \\
 112(168)1 \\
 \underline{112} \\
 56(112)2 \\
 \underline{112} \\
 (0)
 \end{array}$$

Required

Required the greatest common measure to 147 and 323.

$$\begin{array}{r}
 147)323(2 \\
 \underline{294} \\
 29)147(5 \\
 \underline{145} \\
 2)29(14 \\
 \underline{2} \\
 9 \\
 \underline{8} \\
 (1)
 \end{array}$$

As 1 remains in this last question, therefore these two numbers have no common measure.

RULE 6. To abbreviate fractions, *i. e.* to reduce them to their lowest terms; divide both numerator and denominator by their greatest common divisor; the two quotes make the new fraction.

### EXAMPLES.

Abbreviate  $\frac{784}{952}$  to its lowest terms. Note, 56 was found to be the greatest common measure.

$$\begin{array}{r}
 56)784(14 \\
 \underline{56} \\
 224 \\
 \underline{224} \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 56)952(17 \\
 \underline{56} \\
 392 \\
 \underline{392} \\
 \hline
 \end{array}$$

Ans.  $\frac{784}{952} = \frac{14}{17}$

† G

Abbreviate

Abbreviate  $\frac{208}{624}$  to its lowest terms.

$$208)684(3 \quad 4)208(52 \quad 4)684(171 \quad \text{Ans. } \frac{208}{624} = \frac{1}{3}$$

$$\begin{array}{r} \underline{624} \qquad \underline{20} \qquad \underline{4} \\ 6 \cancel{0} 208(3 \quad 8 \qquad 28 \\ \underline{180} \quad 8 \qquad 28 \\ \hline 28)60(2 \qquad 4 \\ \underline{56} \qquad 4 \\ \hline 4)28(7 \\ \underline{28} \\ \hline \end{array}$$

Or thus :

Divide both numerator and denominator by their least common divisor; and you have your fraction in lower terms; after the same manner you may reduce that new fraction to lower terms, and so on continually till no common divisor is found; and you have your fraction in its lowest terms.

### EXAMPLES.

Reduce  $\frac{468}{848}$  to its lowest terms.

$$\frac{1}{2})\frac{468}{848} = \frac{1}{2})\frac{234}{424} (= \frac{1}{2})\frac{117}{212} (= \frac{1}{2})\frac{117}{212} \quad \text{Ans. } \frac{1}{2}$$

N. B. 1. If 5 is on the right hand of both numerator and denominator, or 5 in the one and 0 in the other, 5 measures both.

$$\text{Thus } \frac{5}{5})\frac{105}{105} = \frac{1}{1} \text{ and } \frac{5}{5})\frac{45}{90} = \frac{1}{2}$$

N. B. 2. If there are ciphers on the right hand both of numerator and denominator, cut off an equal

equal number of ciphers from both, and you have your fraction in lower terms.

Thus,  $\frac{8}{60}, \frac{80}{60} = \frac{8}{6} = \frac{4}{3}$

**RULE 7.** To reduce fractions of different denominators to a common denominator : Multiply the denominators continually for the common denominator, and then multiply each numerator into every denominator, except its own, for its corresponding numerator.

### EXAMPLES.

Reduce  $\frac{3}{5}$  and  $\frac{4}{3}$  to a common denominator.

$$\begin{array}{r} 3 \\ 5 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ 5 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ 3 \\ \hline \end{array}$$

15 c. d.    10 n.    12 n. so  $\frac{3}{5} = \frac{12}{15}, \frac{4}{3} = \frac{20}{15}$

Reduce  $\frac{3}{5}, \frac{4}{7}, \frac{7}{9}$  to a common denominator.

$3 \times 5 \times 9 = 135$  c. d.     $2 \times 5 \times 9 = 90$  n.     $4 \times 3 \times 9 = 108$  n.  
 $7 \times 5 \times 3 = 105$  n.

Therefore  $\frac{3}{5}, \frac{4}{7}, \frac{7}{9} = \frac{81}{135}, \frac{108}{135}, \frac{105}{135}$

**N. B.** If the last denominator measures exactly all the preceding denominators, multiply both numerator and denominator of the preceding fractions by the number of times the greater denominator contains the lesser, and your fractions are of a common denominator.

# 76      R E D U C T I O N, &c.

Reduce  $\frac{4}{8}$ ,  $\frac{7}{9}$ ,  $\frac{17}{18}$  to a common denominator.

Thus     $\frac{4}{8}$ ,  $\frac{7}{9}$ ,  $\frac{17}{18} = \frac{13}{18}$ ,  $\frac{14}{18}$ ,  $\frac{17}{18}$

For the last denominator 18 contains the first 6 three times, therefore the first fraction  $\frac{4}{8}$  being multiplied by 3 gives  $\frac{12}{18}$ ; and 18 containing 9 twice, multiply the second fraction  $\frac{7}{9}$  by 2, and you have  $\frac{14}{18}$ , and the last fraction  $\frac{17}{18}$  continues still the same.

We shall annex here some examples on the preceding rules for practice.

Reduce  $\frac{7}{8}$ ,  $\frac{4}{9}$ ,  $\frac{5}{10}$ , and  $\frac{6}{7}$  to a common denominator.    *Ans.*  $\frac{3150}{3150}$ ,  $\frac{1400}{3150}$ ,  $\frac{1575}{3150}$ ,  $\frac{2880}{3150}$ .

Reduce  $\frac{3}{8}$ ,  $\frac{5}{9}$ ,  $\frac{2}{7}$ , and  $\frac{1}{3}$  to a common denominator.    *Ans.*  $\frac{720}{2160}$ ,  $\frac{1200}{2160}$ ,  $\frac{320}{2160}$ ,  $\frac{720}{2160}$ .

Reduce  $\frac{30}{33}$  to its lowest terms.    *Ans.*  $\frac{10}{11}$ .

Reduce  $\frac{12}{16}$  to its lowest terms.    *Ans.*  $\frac{3}{4}$ .

Reduce  $18\frac{1}{2}$  to an improper fraction.    *Ans.*  $\frac{37}{2}$ .

Reduce  $56\frac{1}{2}$  to an improper fraction.    *Ans.*  $\frac{113}{2}$ .

Reduce  $1\frac{1}{2}$  to a mixt number.    *Ans.*  $56\frac{1}{2}$ .

Reduce  $1\frac{1}{2}$  to a mixt number.    *Ans.*  $18\frac{1}{2}$ .

Reduce  $\frac{2}{3}$  of  $\frac{1}{2}$  of  $\frac{5}{8}$  to a simple fraction.    *Ans.*  $\frac{5}{24}$ .

Reduce  $\frac{5}{9}$  of  $\frac{4}{7}$  of  $\frac{11}{12}$  to a simple fraction.    *Ans.*  $\frac{55}{126}$ .

Reduce  $\frac{7}{8}$  of a penny to the fraction of a pound.    *Ans.*  $\frac{7}{16}$  of  $\frac{1}{12}$  of  $\frac{1}{20} = \frac{7}{360}$  l.

Reduce

## REDUCTION of DECIMALS. 77

Reduce  $\frac{4}{7}$  of a penny-weight to the fraction of a lb. Troy. *Ans.*  $\frac{4}{1280}$

Reduce  $\frac{7}{1910}$  of a pound to the fraction of a penny. *Ans.*  $\frac{7}{8}$  of a penny.

For  $7 \times 20 \times 12 = 1680$ , and  $\frac{1680}{1910}$  reduced to the lowest terms is  $\frac{7}{8}$  d.

What is the value of  $\frac{2}{7}$  of a shilling?  
*Ans.* 4 d. 3 f.  $\frac{2}{7}$

## REDUCTION of DECIMALS.

**D**ECIMAL fractions are such as have 10, 100, 1000, 10000, or unity, with any number of ciphers annexed to it, for denominator.

Thus  $\frac{6}{10}$ ,  $\frac{25}{100}$ ,  $\frac{785}{1000}$ ,  $\frac{8}{10000}$ , are decimal fractions; and may be otherwise written thus, .6, .25, .785, .0008; where the point on the left hand, called the decimal point, represents unity (the only significant figure) in the denominator; and the number of figures on the right hand of that point, called the number of decimal places, shews the number of ciphers belonging to the denominator.

**N B.** As in integers, ciphers on the left hand of significant figures do not change their value; so in decimals, ciphers on the right hand of significant figures do not change the value of a decimal, and *e contra*.

Decimals are either finite or infinite; and if infinite, are either infinite repeating, or infinite circulating.

Thus  $\frac{3}{4} = .75$  a finite decimal, and  $\frac{1}{3} = .9333$  an infinite repeating decimal; again  $\frac{1}{8} = .53, 571428, 571428$  an infinite circulating decimal. The circulating

## 78 REDUCTION of DECIMALS.

lating figures begin with 571, &c. which we generally distinguish with a comma, as above?

**RULE 1.** To reduce vulgar fractions to decimals.

Divide the numerator by the denominator, (annexing ciphers to the numerator till it is a sufficient dividend); the quotient is the numerator of the decimal, and the number of ciphers annexed, gives the number of decimal places.

### EXAMPLES.

Reduce  $\frac{3}{4}$  to a decimal.

$$\begin{array}{r} 4 \overline{) 30} \cdot 75 \quad \text{Ans. } \frac{3}{4} = .75 \\ \underline{28} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

Reduce  $\frac{1}{848}$  to a decimal.

$$\begin{array}{r} 848 \overline{) 3000} \cdot 0035377 \\ \underline{2544} \\ 4560 \\ \underline{4240} \\ 3200 \\ \underline{2544} \\ 6560 \\ \underline{5936} \\ 6240 \\ \underline{5936} \\ 304 \end{array}$$

Reduce

# REDUCTION of DECIMALS. 79

Reduce  $\frac{14}{15}$  to a decimal.

$$\begin{array}{r}
 15 \overline{) 140} \quad (.9333 \\
 \underline{135} \phantom{00} \\
 50 \phantom{00} \\
 \underline{45} \phantom{00} \\
 50 \phantom{00} \\
 \underline{45} \phantom{00} \\
 50 \phantom{00} \\
 \underline{45} \phantom{00} \\
 (5)
 \end{array}$$

Reduce  $\frac{1}{9}$  to a decimal.

$$\begin{array}{r}
 9 \overline{) 10} \quad (.111 \\
 \underline{9} \phantom{00} \\
 10 \phantom{00} \\
 \underline{9} \phantom{00} \\
 10 \phantom{00} \\
 \underline{9} \phantom{00} \\
 (1)
 \end{array}$$

Reduce  $\frac{2}{11}$  to a decimal. *Ans.* .32, 142857, 14

Reduce  $\frac{3}{11}$  to a decimal. *Ans.* .27, 27, 27.

What

## 80 REDUCTION of DECIMALS.

What is the decimal of 16 s. 6 d. a pound being the integer?

*s. d.*

$$16 \text{ } 6 = \frac{198}{100}$$

$$240) 1980 (.825$$

$$\underline{1920}$$

$$600$$

$$\underline{480}$$

$$1200$$

$$\underline{1200}$$

What is the decimal of 18 s. 4 d. a pound being the integer?

*Ans. s. d.*

$$18 \text{ } 4 = \frac{220}{100} = .91666$$

What is the decimal of 15 lb. a quarter of an hundred being the integer?

*Ans. lb.*

$$15 = \frac{5}{8} = .53,571428,57, \text{as may be seen above.}$$

**RULE 2.** To continue any division to a decimal.

After your dividend is exhausted, annex three, or if you please more ciphers, and continue your division as before, remembering to set a decimal point in your quotient, when you begin with the ciphers.

Thus,

# REDUCTION of DECIMALS. 31

Thus, 79689 l. divided among 24 men, gives each 3320.375

$$24) 79689 (3320.375$$

$$\begin{array}{r}
 72 \\
 \hline
 76 \\
 72 \\
 \hline
 48 \\
 48 \\
 \hline
 90 \\
 72 \\
 \hline
 180 \\
 168 \\
 \hline
 120 \\
 120 \\
 \hline
 \hline
 \end{array}$$

**RULE 3.** To value decimals in the known parts of the integer.

Multiply the decimal by the number of known parts contained in the integer, and from the product cut off as many decimal places as are in the decimal given; so you have the parts on the left hand of the point; and on the right hand a decimal of one of these parts.

**E X.**

## 82 REDUCTION of DECIMALS.

### EXAMPLES.

What is the value of .876 of a pound ?

$$\begin{array}{r}
 20 \\
 \hline
 17.520 \\
 12 \\
 \hline
 .6240 \\
 4 \\
 \hline
 (.960)
 \end{array}
 \quad
 \begin{array}{l}
 \text{Ans. } \text{£. s. d.} \\
 .876 = 17 \text{ } 6
 \end{array}$$

What is the value of .648 of a Cwt. ?

$$\begin{array}{r}
 4 \\
 \hline
 2.592 \\
 28 \\
 \hline
 4736 \\
 1184 \\
 \hline
 16.576 \\
 16 \\
 \hline
 3456 \\
 576 \\
 \hline
 (9.216)
 \end{array}
 \quad
 \begin{array}{l}
 \text{Cwt. grs. lb. oz.} \\
 \text{Ans. } .648 = 2 \text{ } 16 \text{ } 9
 \end{array}$$

**RULE 4.** To value decimals of a pound by inspection.

Double the figure next the point gives shillings: divide the two following figures (if less than 25) by 4, the quotient gives pence, the remainder farthings: but if these two figures are 25, or more, deduce one before you divide by 4.

E X-

# REDUCTION of DECIMALS. 83

## EXAMPLES.

£.	s.	d.	f.
.718=14	4	2	
.625=12	6	5	
.843=16	10	2	
.509=10	2	1	
.036=0	8	3	
.92=18	5	0	
.7=14	0	0	

N. B. If the figure in the second place from the point is 5, or more than 5, add 1 to your shillings; reject 5 in the second place, and conceive the excess above 5 in the second place, as tens before the figure in the third place; and then find your pence and farthings as before.

## EXAMPLES.

Thus,	£.	s.	d.	f.
	.759=15	2	1	
	.875=17	6	0	
	.95=19	0	0	
	.072=1	5	2	

RULE 5. To reduce shillings, pence, and farthings to a decimal, by inspection; the decimal being only three figure deep. The half of your shillings gives your figure next the point. Multiply your pence by 4; to the product (if less than 24) add your farthings for the two subsequent figures: but if the product is 24 or more, add to it 1 before you add your farthings. N. B. This rule is just the reverse of the preceding one.

E X-

## §4 REDUCTION of DECIMALS.

### EXAMPLES.

Thus,

s.	d.	f.	£.
16	4	2=.	818
12	8	3=.	636
6	2	0=.	308
8	3	1=.	413
0	1	1=.	005

N. B. If the number of shillings is odd, take half of the next lesser number for your figure next the point, and add to the decimal found as before, 050, or 5 to the second figure from the point for the odd shilling.

### EXAMPLES.

Thus,

s.	d.	f.	£.
15	2	1=.	759
17	6	2=.	877
13	4	0=.	666
19	0	0=.	95
1	3	0=.	062

## PRACTICE.

**R**ULES for practice naturally follow the doctrine of fractions: and having previously laid down the rules for reduction of vulgar and decimal fractions, and as a variety of examples has been annexed to illustrate these several rules;

we

we now propose to treat of Practice, prior to single and compound Proportion, which requires more judgment to comprehend its meaning and structure, than any other rule in arithmetic; whereas Practice depends mostly upon division, and the application of the following examples.

The aliquot parts of a pound, with their equivalent simple fractions, stand as below.

s.	d.	℥.
1	: 0	= $\frac{1}{20}$
2	: 0	= $\frac{1}{10}$
2	: 6	= $\frac{1}{8}$
3	: 4	= $\frac{1}{6}$
4	: 0	= $\frac{1}{5}$
5	: 0	= $\frac{1}{4}$
6	: 8	= $\frac{1}{3}$
10	: 0	= $\frac{1}{2}$
13	: 4	= $\frac{2}{3}$
15	: 0	= $\frac{3}{4}$

The aliquot parts of a shilling, or of a pound, with their equivalent simple fractions, stand thus.

d.	s.	℥.
1	= $\frac{1}{12}$	or $\frac{1}{48}$
1½	= $\frac{1}{8}$	= $\frac{1}{24}$
2	= $\frac{1}{6}$	= $\frac{1}{12}$
3	= $\frac{1}{4}$	= $\frac{1}{8}$
4	= $\frac{1}{3}$	= $\frac{1}{6}$
6	= $\frac{1}{2}$	= $\frac{1}{3}$
8	= $\frac{2}{3}$	= $\frac{1}{2}$
9	= $\frac{3}{4}$	= $\frac{1}{2} + \frac{1}{8}$
10	= $\frac{1}{2} + \frac{1}{3}$	= $\frac{5}{6}$

The whole doctrine of Practice will be obvious from the subsequent

## E X A M P L E S.

What cost 87986 lb. of cotton, at 1 s. per lb. ?

$$\begin{array}{r} s \\ 1 = \frac{1}{20} \end{array}$$

$$2,0)8798,6$$

$$\begin{array}{r} \hline \text{£. } 4399 : 6 \\ \dagger \text{ H} \end{array}$$

What

What cost 7648 yards, at 2 s. per yard?

$$\begin{array}{r} s \\ 2 = \frac{1}{10} \end{array}$$

$$1,0)764,8$$

$$\underline{\hspace{1cm}} \\ \text{£. } 764 : 16$$

What cost 7642 lb. of tea, at 2 s. 6 d. per lb.?

$$\begin{array}{r} s \quad d \\ 2 : 6 = \frac{1}{8} \end{array}$$

$$8)7642$$

$$\underline{\hspace{1cm}} \\ \text{£. } 955 : 5$$

What cost 3468 yards of linen, at 3 s. 4 d. per yard?

$$\begin{array}{r} s. \quad d. \\ 3 \quad 4 = \frac{1}{6} \end{array}$$

$$6)3468$$

$$\underline{\hspace{1cm}} \\ \text{£. } 578$$

What cost 796 yards of cloth, at 4 s. per yard?

$$\begin{array}{r} s. \\ 4 = \frac{1}{5} \end{array}$$

$$5)796$$

$$\underline{\hspace{1cm}} \\ \text{£. } 159 : 4$$

What cost 868 Scots quarts of spirits, at 5 s. per quart?

$$\begin{array}{r} s. \\ 5 = \frac{1}{4} \end{array}$$

$$4)868$$

$$\underline{\hspace{1cm}} \\ \text{£. } 217$$

What

What cost 1768 lb. of tea, at 6 s. 8 d. per lb.?

$$\begin{array}{r}
 \text{s.} \quad \text{d.} \\
 6 \quad 8 = \frac{2}{3} \\
 3) 1768 \\
 \hline
 \text{£. } 589 : 6 : 8
 \end{array}$$

What cost 847 yards of broad cloth, at 10 s. per yard?

$$\begin{array}{r}
 \text{s.} \\
 10 = \frac{1}{2} \\
 2) 847 \\
 \hline
 \text{£. } 423 : 10
 \end{array}$$

What cost 1768 lb. of rhubarb, at 13 s. 4 d. per lb.?

$$\begin{array}{r}
 \text{s.} \quad \text{d.} \\
 13 \quad 4 = \frac{1}{3} \\
 1768 \\
 \quad 2 \\
 \hline
 3) 3536 \\
 \hline
 \text{£. } 1178 : 13 : 4
 \end{array}
 \quad
 \begin{array}{l}
 \text{Or thus, } 3) 1768 \\
 \hline
 589 \quad 6 \quad 8 \\
 589 \quad 6 \quad 8 \\
 \hline
 \text{£. } 1178 : 13 : 4
 \end{array}$$

What cost 844 yards of broad cloth, at 15 s. per yard?

$$\begin{array}{r}
 \text{s.} \\
 15 = \frac{1}{3} \\
 844 \\
 \quad 3 \\
 \hline
 4) 2532 \\
 \hline
 \text{£. } 633
 \end{array}
 \quad
 \begin{array}{l}
 \text{Or thus, } 4) 844 \\
 \hline
 211 \\
 211 \\
 211 \\
 \hline
 \text{£. } 633
 \end{array}$$

N. B. If the price of the unit is any even number

ber of shillings, different from 2, 4, 10, multiply  $\frac{1}{2}$  of the price into the sum given, always doubling the excess of the product of the right-hand figure for shillings, and carry the tens into the product of the immediately following place of pounds.

## EXAMPLES.

What is the price of 4323 yards, at 6 s. per yard?

$$\begin{array}{r} 4323 \\ 3 \\ \hline \end{array}$$

£. 1296 : 18

What is the price of 4768 lb. of tea, at 8 s. per lb.?

$$\begin{array}{r} 4768 \\ 4 \\ \hline \end{array}$$

£. 1907 : 4

What is the price of 6478 yards of broad cloth, at 18 s. per yard?

$$\begin{array}{r} 6478 \\ 9 \\ \hline \end{array}$$

£. 5830 : 4

N. B. If the price of the unit of the wares is any odd number of shillings, different from 5 or 15, work for the next even number of shillings,  
as

as before, and for the odd shilling take  $\frac{1}{20}$  of the given sum.

## EXAMPLES.

If 1 yard cost 17 s. what will 859 yards cost?

$$\begin{array}{r}
 859 \\
 \underline{8} \\
 687 \quad 4 \\
 \frac{1}{20})859 = 42 \quad 19 \\
 \underline{\hspace{1.5cm}} \\
 \text{£. } 73^{\circ} \quad 3
 \end{array}$$

If 1 yard of broad cloth cost 19 s. what will 8462 yards cost?

$$\begin{array}{r}
 8462 \\
 \underline{9} \\
 7615 \quad 16 \quad \text{Or thus, } 8462 \\
 \frac{1}{20})8462 = 423 \quad 2 \quad \frac{1}{20} = 423 \quad 2 \\
 \underline{\hspace{1.5cm}} \quad \underline{\hspace{1.5cm}} \\
 \text{£. } 8038 \quad 18 \quad \text{£. } 8038 : 18
 \end{array}$$

N. B. The following examples have the price of one of the commodities, the aliquot parts of a shilling.

H 3

E X

## EXAMPLES.

At 1 d. per lb. what cost 9764 lb. ?

*d.*

$$1 = \frac{1}{1} \text{ or } \frac{1}{1} \quad 2)9764 \text{ Or thus, } 24,0)976,4$$


---


$$2,0)81,3 \quad 8 \quad \text{£. } 40 : 13 : 8$$


---


$$\text{£. } 40 : 13 : 8$$

At  $1\frac{1}{2}$  d. per lb. what cost 7968 lb. ?

$$1\frac{1}{2} = \frac{3}{2} \text{ or } \frac{3}{2} \quad 8)7968 \text{ Or thus, } 16,0)796,8$$


---


$$2,0)99,6 \quad \text{£. } 49 : 16$$


---


$$\text{£. } 49 : 16$$

At 2 d. per lb. what cost 13147 lb. of raisins ?

*d.*

$$2 = \frac{2}{1} \text{ or } \frac{2}{1} \quad 6)13147 \text{ Or thus, } 12,0)1314,7$$


---


$$2,0)219,1 \quad 2 \quad \text{£. } 109 : 11 : 2$$


---


$$10 : 11 : 2$$

At 3 d. per lb. what cost 87341 lb. of sugar ?

*d.*

$$3 = \frac{3}{1} \text{ or } \frac{3}{1} \quad 4)87341 \text{ Or thus, } 8,0)8734,1$$


---


$$2,0)2183,5 \quad 3 \quad \text{£. } 1091 : 15 : 3$$


---


$$\text{£. } 1091 : 15 : 3$$

At

At 4 d. per lb. what cost 3097 lb. of figs?

$$4 \text{ d.} = \frac{1}{3} \text{ or } \frac{2}{6}$$

$$3)3097 \text{ Or thus, } 6,0)309,7$$

$$\begin{array}{r} 2,0)103,2 \quad 4 \\ \hline \end{array}$$

$$\text{£. } 51 : 12 : 4$$

$$\text{£. } 51 : 12 : 4$$

At 6 d. per lb. what cost 78642 lb. of sugar?

$$6 \text{ d.} = \frac{1}{2} \text{ or } \frac{1}{2}$$

$$2)78642$$

$$\text{Or thus, } 4,0)7864,2$$

$$\begin{array}{r} 2,0)3932,1 \\ \hline \end{array}$$

$$\text{£. } 1966 : 1$$

$$\text{£. } 1966 : 1$$

N. B. If the price of unity, or one integer of any commodity, is not an aliquot part of a shilling, (as 5, 7, 8, 9, 10, or 11), you may either resolve it into such parts as are aliquot parts of a shilling, or such as are aliquot parts of a pound, or some just divisor of 20 shillings, or a pound.

## EXAMPLES.

At 5 d. per lb. what cost 3071 lb. of figs?

d. d. d.

$$5 = 3 + 2 = \frac{1}{2} + \frac{1}{2}$$

$$4)3071$$

$$\begin{array}{r} 767 \quad 9 \\ \hline \end{array}$$

$$\frac{1}{2} = 511 \quad 10$$

$$\begin{array}{r} 2,0)127,9 \quad 7 \\ \hline \end{array}$$

$$\text{£. } 63 : 19 : 7$$

Or thus,

d.

$$5 = \frac{1}{2} = 8 \times 6$$

$$8)3071$$

$$\begin{array}{r} 6)383 \quad 17 \quad 6 \\ \hline \end{array}$$

$$\text{£. } 63 : 19 : 7$$

At

At 7 d. per lb. what cost 321 lb. of any thing?

*d. d. d.*

$$\begin{array}{r}
 7 = 3 + 4 = \frac{1}{2} + \frac{1}{3} \\
 4 \overline{) 321} \\
 \underline{\phantom{00}80} \phantom{3} \\
 \frac{1}{3} = 107 \\
 2,0 \overline{) 187 \phantom{3}} \\
 \underline{\phantom{00}00} \\
 \text{£. } 9 : 7 : 3
 \end{array}$$

Or thus,

*d.*

$$\begin{array}{r}
 7 = \frac{1}{2} + \frac{1}{3} \\
 6,0 \overline{) 32,1} \\
 \underline{\phantom{00}57} \\
 \frac{1}{3} = 4 \phantom{0} 3 \\
 \underline{\phantom{00}00} \\
 \text{£. } 9 : 7 : 3
 \end{array}$$

At 8 d. per yard, what cost 3746 yards of ribbon?

*d.*

$$8 = \frac{2}{3} \text{ or } \frac{1}{30}$$

3746

Or thus,

$$3,0 \overline{) 374,6}$$

$$\begin{array}{r}
 3746 \\
 \underline{\phantom{00}2} \\
 3 \overline{) 7492} \\
 \underline{\phantom{00}00} \\
 2,0 \overline{) 249,7 \phantom{4}} \\
 \underline{\phantom{00}00} \\
 \text{£. } 124 : 17 : 4
 \end{array}$$

$$\text{£. } 124 : 17 : 4$$

At

At 9 d. per lb. what cost 4052 lb. of sugar?

$$\begin{array}{r}
 d. \cdot d. \cdot d. \\
 9 = 6 + 3 = \frac{1}{2} + \frac{1}{4} \\
 2) 4052 \\
 \hline
 2026 \\
 \frac{1}{4} = 1013 \\
 \hline
 2,0) 303,9 \\
 \hline
 \pounds. 151 : 19
 \end{array}$$

Or thus,

$$\begin{array}{r}
 d. \\
 9 = \frac{1}{2} \text{ of } \frac{3}{4} \\
 4,0) 405,2 \\
 \hline
 101 \quad 6 \\
 \frac{1}{4} = 50 \quad 13 \\
 \hline
 \pounds. 151 : 19
 \end{array}$$

At 10 d. per lb. what cost 3179 lb. of black sugar?

$$\begin{array}{r}
 d. \cdot d. \cdot d. \\
 10 = 6 + 4 = \frac{1}{2} + \frac{1}{4} \\
 2) 3179 \\
 \hline
 1589 \quad 6 \\
 \frac{1}{4} = 1059 \quad 8 \\
 \hline
 2,0) 264,9 \quad 2 \\
 \hline
 \pounds. 132 : 9 : 2
 \end{array}$$

Or thus,

$$\begin{array}{r}
 d. \\
 10 = \frac{1}{2} + \frac{1}{4} \\
 4,0) 317,9 \\
 \hline
 79 \quad 9 \quad 6 \\
 \frac{1}{4} = 52 \quad 19 \quad 8 \\
 \hline
 \pounds. 132 : 9 : 2
 \end{array}$$

If the price is 11 d. take  $\frac{1}{2}$  of the sum from it, and you have the price in shillings.

At

At 11 d. per lb. what cost 7642 lb. of any thing?

$$\begin{array}{r}
 12 \overline{) 7642} \\
 \underline{636} \ 10 \\
 2,0 \overline{) 700,5} \ 2 \\
 \underline{\phantom{2,0} 15,1} \ 2 \\
 \underline{\phantom{2,0} 15,1} \phantom{2} \\
 \phantom{2,0} 0
 \end{array}$$

$\pounds. 350 : 5 : 2$

If the price is farthings under a penny, find what fraction it is of a penny, or of a shilling, and find the value of your goods in pence or shillings, which are easily reduced to pounds.

### EXAMPLES.

At 1 farthing per yard, what cost 7258 yards of tape?

$$\begin{array}{r}
 4 \overline{) 7258} \\
 \underline{12} \ 1814 \ 2 \\
 2,0 \overline{) 15,1} \ 2 \\
 \underline{\phantom{2,0} 15,1} \phantom{2} \\
 \phantom{2,0} 0
 \end{array}$$

$\pounds. 7 : 11 : 2 : 2$

Or thus,  $1 \text{ f.} = \frac{1}{4} \text{ p.} = \frac{1}{16} \text{ of } 1 \text{ s.}$

$$12,0 \overline{) 725,8}$$

$$8 \overline{) 60} \ 9 \ 8$$

$$\pounds. 7 : 11 : 2 : 2$$

At

At 2 farthings per yard, what cost 38746 yards?

$$\begin{array}{r}
 2)38746 \\
 \hline
 12)19373 \\
 \hline
 2,0)161,4 \quad 5 \\
 \hline
 \text{£. } 80 : 14 : 5
 \end{array}
 \quad
 \begin{array}{l}
 \text{Or thus, } 2 \text{ f.} = \frac{1}{2} = \frac{1}{4} \text{ of } \frac{1}{16} \\
 12,0)3874,6 \\
 \hline
 4)322 \quad 17 \quad 8 \\
 \hline
 \text{£. } 80 : 14 : 5
 \end{array}$$

At 3 farthings per yard, what cost 56086 yards?

$$\begin{array}{r}
 3 \text{ f.} = \frac{1}{2} + \frac{1}{4} \\
 2)56086 \\
 \hline
 28043 \\
 \frac{1}{4} = 14021 \quad 2 \\
 \hline
 12)42064 \quad 2 \\
 \hline
 2,0)350,5 \quad 4 \quad 2 \\
 \hline
 \text{£. } 175 : 5 : 4 : 2
 \end{array}$$

*f. d.*

Or thus,  $4 \times 3 = 12$  and  $80 \times 3 = 240 = 1 \text{ l.}$   
 Therefore take  $\frac{1}{12}$  of  $\frac{1}{4}$  of the sum.

$$\begin{array}{r}
 4)56086 \\
 \hline
 8,0)1402,1 \quad 1 \frac{1}{2} \\
 \hline
 \text{£. } 175 : 5 : 4 \frac{1}{2}
 \end{array}$$

If

If the price of the integer is shillings and pence, and not an aliquot part of a pound, divide the price into such parts, as at least one of these parts shall be an aliquot part of a pound; work for that part, as directed in the rules for the aliquot parts of a pound, and with the other part or parts, as directed in the rules for shillings and pence.

## EXAMPLES.

At 3 s. 10 d. per yard, what cost 796 yards?

3 s. 10 d. = 3 s. and 4 d. + 6 d. =  $\frac{1}{8}$  +  $\frac{1}{16}$  of a l.

6)796 Or thus, 3 s. 10 =  $\frac{1}{8}$  +  $\frac{1}{16}$  +  $\frac{1}{32}$  of  $\frac{1}{16}$

$$\begin{array}{r} 132 \quad 13 \quad 4 \\ \frac{1}{16} = 19 \quad 18 \\ \hline \text{£. } 152 : 11 : 4 \end{array}$$

$$\begin{array}{r} 8)796 \\ \hline 99 \quad 10 \\ \frac{1}{16} = 39 \quad 16 \\ \frac{1}{32} = 13 \quad 5 \quad 4 \\ \hline \text{£. } 152 : 11 : 4 \end{array}$$

At 17 s. 4 d. per yard, what cost 394 yards?

17 : 6 = 6 : 8 + 6 : 8 + 4 =  $\frac{1}{3}$  +  $\frac{1}{3}$  +  $\frac{1}{3}$

Or thus,

$$\begin{array}{r} 3)394 \\ \hline 131 \quad 6 \quad 8 \\ 131 \quad 6 \quad 8 \\ \frac{1}{3} = 78 \quad 16 \\ \hline \text{£. } 341 : 9 : 4 \end{array}$$

$$\begin{array}{r} 394 \\ 8 \\ \hline \text{at } 16 \text{ s.} = 315 \quad 4 \\ \text{at } 1 \text{ s.} = \frac{1}{16} = 19 \quad 14 \\ 4 \text{ d.} = \frac{1}{3} = 6 \quad 11 \quad 4 \\ \hline \text{£. } 341 : 9 : 4 \end{array}$$

At

# PRACTICE.

97

At 14 s. 10 d. per yard, what cost 1904 yards?

$$14 : 10 = 13 \text{ s. } 4 \text{ d. } + 1 \text{ s. } + 6 \text{ d. } = \frac{1}{3} + \frac{1}{10} + \frac{1}{5} \text{ or } \frac{1}{2} \text{ of } \frac{1}{5}$$

$\begin{array}{r} 3) 1904 \\ \hline 634 \text{ } 13 \text{ } 4 \\ 634 \text{ } 13 \text{ } 4 \\ \hline 1 \frac{1}{10} = 95 \text{ } 4 \\ \frac{1}{2} = 47 \text{ } 12 \\ \hline \end{array}$	<p>Or thus,</p> $\begin{array}{r} 1904 \\ \hline 7 \\ \hline \end{array}$ <p>at 14 s. = 1332 16 6 d. <math>\frac{1}{5}</math> = 47 12 4 d. <math>\frac{1}{10}</math> = 31 14 8 <hr/> </p>
<p>£. 1412 : 2 : 8</p>	<p>£. 1412 : 2 : 8</p>

At 19 s. 9 d. per yard, what cost 1504 yards of broad cloth?

s. s. s. s. d. d.

$$19 : 9 = 5 + 5 + 5 + 4 + 8 + 1 = \frac{1}{4} \times 3 + \frac{1}{5} + \frac{1}{10} \text{ of } \frac{1}{5} + \frac{1}{10} \text{ of } \frac{1}{5}$$

$\begin{array}{r} 4) 1504 \\ \hline 376 \\ 3 \\ \hline 1128 \\ \frac{1}{5} = 300 \text{ } 16 \\ \frac{1}{5} \text{ of } \frac{1}{5} = 50 \text{ } 2 \text{ } 8 \\ \frac{1}{10} \text{ of } \frac{1}{5} = 6 \text{ } 5 \text{ } 4 \\ \hline \end{array}$	<p>Or thus, 19 : 9 = 18 + 1 + 6 + 3</p> $\begin{array}{r} 1504 \\ \hline 9 \\ \hline 18 = 1353 \text{ } 12 \\ 1 = \frac{1}{10} = 75 \text{ } 24 \\ 6 \text{ d. } = \frac{1}{2} = 37 \text{ } 12 \\ 3 = \frac{1}{2} = 18 \text{ } 16 \\ \hline \end{array}$
<p>£. 1485 : 4 : 0</p>	<p>£. 1485 : 4</p>

† I

At

At 2 s. 4 d. per yard, what cost 1865 yards?

$$2 : 4 = \frac{1}{10} + \frac{1}{10} \quad \text{Or thus, } 2 : 4 = \frac{1}{10} + \frac{1}{10} \text{ of } \frac{1}{10}$$

$$\begin{array}{r} 1,0)186,5 \\ \hline 186 \ 10 \\ \frac{1}{10} = \quad 31 \ 1 \ 8 \\ \hline \pounds. 217 : 11 : 8 \end{array}$$

$$\begin{array}{r} 1,0)186,5 \\ \hline 186 \ 10 \\ \frac{1}{10} = \quad 31 \ 1 \ 8 \\ \hline \pounds. 217 : 11 : 8 \end{array}$$

If the price is pence and farthings, work for pence as before; if the farthings are an aliquot part of the pence, take that part of the price of pence; otherwise work for your farthings as before taught, where the price was only farthings.

## EXAMPLES.

At  $3\frac{1}{2}$  d. per dozen, what cost 3471 dozen of buttons?

$$\begin{array}{r} 4)3471 \\ \hline 867 \ 9 \\ 2 \ f = \frac{1}{8} \quad 144 \ 7\frac{1}{2} \\ \hline 2,0)101,2 \ 4\frac{1}{2} \\ \hline \pounds. 50 : 12 : 4\frac{1}{2} \end{array}$$

$$\begin{array}{r} d. \\ \text{Or thus, } 3 = \frac{1}{80})3471 \\ \hline 43 \ 7 \ 9 \\ \frac{1}{80} = \quad 7 \ 4 \ 7\frac{1}{2} \\ \hline \pounds. 50 : 12 : 4\frac{1}{2} \end{array}$$

At

At  $5\frac{1}{4}$  d. per lb. what cost 9761 lb. of sugar?

$5\frac{1}{4}$  d. = 3 + 2 + 1 f. Or thus,  $5\frac{1}{4}$  d. =  $\frac{1}{80} + \frac{1}{110} + \frac{1}{8}$  of  $\frac{1}{110}$

$$\begin{array}{r}
 4)9761 \\
 \hline
 2440 \quad 3 \\
 \frac{1}{6} = 1626 \quad 10 \\
 \frac{1}{8} \text{ of } \frac{1}{6} = 203 \quad 4 \quad 1 \\
 \hline
 2,0)427,0 \quad 5 \quad 1 \\
 \hline
 \text{£. } 213 : 10 : 5 : 1 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 8,0)976,1 \\
 \hline
 122 \quad 0 \quad 3 \\
 \frac{1}{110} = 81 \quad 6 \quad 10 \\
 \frac{1}{8} \text{ of } \frac{1}{110} = 10 \quad 3 \quad 4\frac{1}{4} \\
 \hline
 \text{£. } 213 : 10 : 5\frac{1}{4} \\
 \hline
 \end{array}$$

At  $3\frac{3}{4}$  d. per lb. what cost 1794 lb. of pepper?

$3\frac{3}{4}$  d. =  $\frac{1}{4} + \frac{1}{4}$  of  $\frac{1}{4}$  Or thus,  $3\frac{3}{4}$  d. =  $\frac{1}{80} + \frac{1}{4}$  of  $\frac{1}{80}$

$$\begin{array}{r}
 4)1794 \\
 \hline
 448 \quad 6 \\
 \frac{1}{4} \text{ of } \frac{1}{4} = 112 \quad 1\frac{1}{2} \\
 \hline
 2,0)56,0 \quad 7\frac{1}{2} \\
 \hline
 \text{£. } 28 : 0 : 7\frac{1}{2} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 8,0)179,4 \\
 \hline
 22 \quad 8 \quad 6 \\
 \frac{1}{4} \text{ of } \frac{1}{80} = 5 \quad 12 \quad 1\frac{1}{2} \\
 \hline
 \text{£. } 28 : 0 : 7\frac{1}{2} \\
 \hline
 \end{array}$$

At  $4\frac{3}{4}$  d. per yard, what cost 3475 yards of ribbon?

$4\frac{3}{4}$  d. =  $\frac{1}{3} + \frac{1}{8}$  of  $\text{£}$  Or thus,  $4\frac{3}{4}$  d. =  $\frac{1}{60} + \frac{1}{8}$  of  $\frac{1}{60}$ .

$$\begin{array}{r}
 3 \overline{) 3475} \\
 \underline{1158} \quad 4 \\
 \frac{1}{6} \text{ of } \frac{1}{3} = 193 \quad \frac{2}{3} \\
 \underline{2,0) 135,1} \quad 4\frac{2}{3} \\
 \underline{\text{£. } 67 : 11 : 4\frac{2}{3}}
 \end{array}$$

$$\begin{array}{r}
 6,0 \overline{) 347,5} \\
 \underline{57 \quad 18 \quad 4} \\
 \frac{1}{6} \text{ of } \frac{1}{60} = 9 \quad 13 \quad \frac{2}{3} \\
 \underline{\text{£. } 67 : 11 : 4\frac{2}{3}}
 \end{array}$$

If the price is pounds, shillings, pence, and farthings, multiply the number given by the pounds, and work for the shillings, pence, and farthings, as before.

### EXAMPLES.

At 2 l. 3 s.  $8\frac{1}{2}$  d. per hundred, what cost 276 Cwt. 2 qrs. 15 lb. of steel?

$$\begin{array}{r}
 276 \\
 \underline{2} \\
 552 \quad \text{at 2 l.} \\
 46 \quad \text{at 3 s. 4. d.} \\
 4 \quad 12 \quad \text{at 4 d} \\
 11 \quad 6 \quad \text{at 2 far.} \\
 \hline
 603 \quad 3 \quad 6 = 276 \text{ Cwt.} \\
 1 \quad 1 \quad 10\frac{1}{2} = 2 \text{ qrs} \\
 5 \quad 5\frac{1}{2} = 14 \text{ lb.} \\
 \hline
 \text{£. } 604 : 10 : 9\frac{1}{2}
 \end{array}$$

At

At 3 l. 18 s. 6 d. per Cwt. what cost 389 Cwt.  
3 qrs. 16 lb. of rhubarb?

Or thus,

$$\begin{array}{r}
 389 \\
 78 \\
 \hline
 3112 \\
 2723 \\
 \hline
 30342
 \end{array}
 \qquad
 \begin{array}{r}
 389 \\
 3 \\
 \hline
 1167 \\
 389 \times 9 = 3502 \\
 \frac{1}{4} = 9146 \\
 3101 = 3 \text{ qr. } 16 \text{ lb.}
 \end{array}
 \qquad
 \begin{array}{l}
 \text{at } 3 \text{ l.} \\
 \text{at } 18 \text{ s.} \\
 \text{at } 6 \text{ d.} \\
 3 \text{ qr. } 16 \text{ lb.}
 \end{array}$$

$$\begin{array}{r}
 30342 \\
 1946 \\
 393 = 2 \text{ qr.} \\
 197\frac{1}{2} = 1 \text{ qr.} \\
 112\frac{1}{2} = 16 \text{ lb.}
 \end{array}
 \left. \begin{array}{l} \\ \\ \\ \end{array} \right\}
 \begin{array}{l}
 \text{£. } 1530 \text{ } 6 \text{ } 7 \\
 \text{£. } \text{ } \text{ } \text{ } \\
 \text{s. } 3 \text{ } 10 \text{ } 1 \\
 \text{d.}
 \end{array}$$

$$\begin{array}{r}
 2,0) 3060,67 \\
 \hline
 1530 \text{ } 6 \text{ } 7
 \end{array}$$

What cost 238 Cwt. 1 qr. 7 lb. at 6 l. 14 s.  
10½ d. per Cwt.?

$$\begin{array}{r}
 238 \\
 134 \\
 \hline
 31892
 \end{array}
 \qquad
 \begin{array}{r}
 \text{qr.} \\
 1 = 1 \text{ } 13 \text{ } 8\frac{1}{2} \\
 7 \text{ lb} = 8 \text{ } 5 \\
 \hline
 2 \text{ } 2 \text{ } 1\frac{3}{4}
 \end{array}$$

$$\begin{array}{r}
 6 \text{ d.} = \frac{1}{2} = 119 \\
 4 = \frac{1}{4} = 79 \text{ } 4 \\
 \frac{1}{8} \text{ of } \frac{1}{4} = 9 \text{ } 11 \\
 42 \text{ } 1\frac{1}{2} \\
 \hline
 2,0) 3214,2 \text{ } 4\frac{1}{2} \\
 \hline
 \text{£. } 1607 : 2 : 4\frac{1}{2}
 \end{array}$$

Or thus, 238  
6

$$\begin{array}{r}
 1428 \\
 \hline
 \frac{1}{3} = 79 \quad 6 \quad 8 \text{ at } 6 \text{ l.} \\
 \frac{1}{3} = 79 \quad 6 \quad 8 \text{ at } 6 \text{ s. } 8 \text{ d.} \\
 \frac{1}{30} = 11 \quad 18 \quad \text{at } 1 \text{ s.} \\
 \frac{1}{3} = 5 \quad 19 \quad \text{at } 6 \text{ d.} \\
 \frac{1}{12} = 9 \quad 11 \text{ at } 2 \text{ far.} \\
 \hline
 2 \quad 2 \quad 1\frac{1}{2} = 1 \text{ qr. } 7 \text{ lb.} \\
 \hline
 1607 \quad 2 \quad 4\frac{1}{2}
 \end{array}$$

## PROPORTION.

**T**HIS rule, from its excellency and general utility, is called by some *the golden Rule*; and by others *the Rule of Three*, because from three numbers given, it finds a fourth proportional.

Under this rule will be comprehended these several rules, *viz.* Fellowship, Interest, Barter, &c. because, after some preparative steps are taken, they may be all reduced to the rule of Proportion, however much pains some authors have taken to exhibit them under so many different names, with a multiplicity of rules for working them annexed.

Proportion is either simple or compound; and it is likewise direct or inverse,

**RULE**

**RULE 1.** To state the question, *i. e.* to place the numbers in the simple rule.

Set that number in the question, which is of the same species with the number sought, in the second place; that which gives it, in the first; and the third sets itself.

**RULE 2.** To know if a question is of the direct or inverse.

If more give more, or less give less; the question is direct. If more give less, or less give more; the question is inverse.

**RULE 3.** To work the direct rule. Multiply the two last numbers together, and divide their product by the first; the quotient gives the answer.

**RULE 4.** To work the inverse rule.

Multiply the two first numbers together, and divide their product by the last; and the quotient gives the answer.

## E X A M P L E S.

If a man gains 2 s. in 3 days, what will he gain in 12 days?

$$\begin{array}{r}
 d. \ s. \ d. \\
 3 \ 2 \ 12 \\
 \qquad \qquad 2 \\
 \hline
 3) \ 24 \\
 \hline
 8 \ s. \ Ans.
 \end{array}$$

If 76 men spend 340 l. in any time, what will  
15 men spend in the same time?

<i>m.</i>	<i>l.</i>	<i>m.</i>
76	340	15
	15	
<hr/>		
	1700	
	340	
<hr/>		
76)	5100	(67.105=67:2:1:1 <i>Ans.</i>
	456	
<hr/>		
	540	
	532	
<hr/>		
	80	
	76	
<hr/>		
	400	
	380	
<hr/>		
	(20)	

N. B. Instead of reducing the remainder 8 to shillings, I annexed 3 ciphers successively, and continuing my division, I found .105, which reduced by inspection, amounts to 2 s. 1 d. 1 far. as above; and thus the work is shortened by no less than 13 figures.

# P R O P O R T I O N. 105

If 36 yards be a rood of mason-work, at 4 quarters high, what will be a rood, at 7 quarters high?

$$\begin{array}{rcl}
 \text{qrs.} & \text{y.} & \text{q.} \\
 4 & : 36 & :: 7 \\
 & 4 & \\
 \hline
 7 & \overline{) 144} & \\
 & 20 : 4 & \text{y. qrs.} \\
 & 4 & \text{Ans. } 20 : 2 \\
 & - & \\
 & 16 & 2 \\
 & 14 & \\
 & - & \\
 & (2) & 
 \end{array}$$

If 36 men do a piece of work in 40 days, in what time will 15 men do the same?

$$\begin{array}{rcl}
 m. & d. & m. \\
 36 & : 40 & :: 15 \\
 & 36 & \\
 15 & \overline{) 1440} & (96 \text{ days } \text{Ans.} \\
 & 135 & \\
 & - & \\
 & 90 & \\
 & 90 & \\
 & - & \\
 & 0 & 
 \end{array}$$

N. B. It will sometimes be necessary to reduce either the first or last term, or both, before you multiply; for if either of the terms be pence or ounces, &c. the other of consequence must be so too, as in the following example.

If

If  $4\frac{3}{4}$  yards cost 5 l. what will 86 yards cost ?

$$\begin{array}{rcl}
 y. & l. & y. \\
 4\frac{3}{4} : 5 :: 86 & & \\
 \hline
 4 & & 4 \\
 \hline
 19 & & 344 \\
 & & \hline
 & & 5
 \end{array}$$

$$\begin{array}{r}
 \text{£.} \\
 19)1720(90.526=90 : 10 : 6 : 1 \text{ Ans.} \\
 \underline{171}
 \end{array}$$

100

95

50

38

120

114

(6)

Three partners, A, B, and C, give in their stocks to trade; A gives 536 l. B gives 320 l. and C 144 l. they gain 288 l: what is each man's share thereof?

$$\begin{array}{rcl}
 \text{£.} & & \\
 \text{A's stock} = & 536 & \\
 \text{B's do} = & 320 & \\
 \text{C's do} = & 144 & \\
 \hline
 \text{Total stock} & 1000 & \\
 \hline
 \end{array}$$

$$\begin{array}{rcl}
 \text{Total st. l. A's st.} & & \\
 1000 : 288 :: 536 & & \\
 & 536 & \\
 \hline
 & 1728 & \\
 & 864 & \\
 & 1440 & \\
 \hline
 \text{£.} & &
 \end{array}$$

$$\begin{array}{r}
 1,000)154,368=154 : 7 : 4\frac{1}{2} \\
 \text{Tot.}
 \end{array}$$

*Tot. ft. l. B's ft. Tot. ft. l. C's ft.*

1000 : 288 :: 320      1000 : 288 :: 144

320

144

5760

1152

864

1152

288

1,000)92,160=92 : 3 : 2 : 2 ———— £

1,000)41,472=41 : 9 : 5 : 2

N. B. It is obvious, that there is above an error of 2 farthings, but this will always happen; for the pound contains 960 farthings, and a decimal of three places divides the pound into 1000 equal parts, which makes an error of 40; and 50 errs in 2, which is the case above; which 2 being deducted, the proof stands thus.

	£.	s.	d.	f.
A's gain=	154	7	4	2
B's ditto=	90	3	2	2
C's ditto=	41	9	5	2
Proof 288	0	0	0	

What is the interest of 642 l. at 5 per cent. *per annum*?

<i>l. l. l.</i>		<i>£.</i>
100 : 5 :: 642	Or thus,	642 principal.
5		5 interest.
—————		—————
1,00)32,10=32 : 2		32,10
		20
		—————
		2,00

N. B.

N. B. In the above and similar questions, multiply the principal by the rate of interest, and point off 2 figures towards the right hand, and those on the left are pounds; and multiply by 20, 12, and 4; and point off 2 figures each time, and you will have your shillings, pence, and farthings on the left hand of the point.

Three persons A, B, and C, hire a pasture for 24 l. A puts in 40 cows for 4 months, B 30 cows for 2 months, and C 36 cows for 5 months: what share of the rent must each person pay?

$  \begin{array}{r}  A \ 40 \times 4 = 160 \\  B \ 30 \times 2 = 60 \\  C \ 36 \times 5 = 180 \\  \hline  400  \end{array}  $	$  \begin{array}{r}  400 : 24 :: 160 \\  \phantom{400 : 24 :: } 24 \\  \hline  640 \\  320 \\  \hline  \text{£.} \\  400 : 24 :: 60 \\  \phantom{400 : 24 :: } 60 \\  \hline  \text{£.} \\  400 : 1440 (3.6 = 3 : 12 \\  \phantom{400 : 1440 (3.6 = 3 : 12} 1200 \\  \hline  2400 \\  2400 \\  \hline  \text{£. s.} \\  \text{Proof, A's share} = 9 \ 12 \\  \phantom{Proof, A's share} B's ditto = 3 \ 12 \\  \phantom{Proof, A's share} C's ditto = 10 \ 16 \\  \hline  \text{£. 24}  \end{array}  $
	$  \begin{array}{r}  4,000 \ 3840 (9.6 = 9 : 12 \\  \phantom{4,000 \ 3840 (9.6 = 9 : 12} 3600 \\  \hline  2400 \\  2400 \\  \hline  \text{£.} \\  400 : 24 :: 180 \\  \phantom{400 : 24 :: } 24 \\  \hline  720 \\  360 \\  \hline  \text{£.} \\  400 : 4320 (10.8 = 10 : 16 \\  \phantom{400 : 4320 (10.8 = 10 : 16} 400 \\  \hline  3200 \\  3200 \\  \hline  \end{array}  $

If I buy hats at 4 s. each, and sell them again at 4 s. 9 d. what profit do I make upon 100 l.?

$$\begin{array}{r}
 \begin{array}{ccc} s & d. & l. \\ 4 & 9 & :: 100 \end{array} \\
 \hline
 20 \\
 \hline
 2,000 \\
 9 \\
 \hline
 4) 18000 \\
 \hline
 12) 4500 \\
 \hline
 2,0) 37,5 \\
 \hline
 \text{£. } 18 : 15 \text{ Ans.}
 \end{array}$$

16 A and B put in stocks for trade; A puts in 40 l. and at 4 months end takes out 10 l. but 2 months after puts in 30 l.; B puts in 50 l. and at the end of 3 months puts in 20 l. more; at the  
 † K end

end of 8 months they have gained 18 l. : what is each man's share of the gain?

$$\begin{array}{rcl}
 \text{A puts in 40 l.} & -10=30 & +30=60 \\
 \text{for 4 m} & 2 & 2 \\
 \hline
 160 & 60 & 120=340 \\
 \text{B puts in 50 l.} & +20 & =70 \\
 \text{for 3 m} & 5 & \\
 \hline
 150 & 350 & = 500 \\
 & & \hline
 & & 840
 \end{array}$$

$$\begin{array}{rcl}
 840:18::340 & 840:18::500 \\
 18 & 18 \\
 \hline
 2720 & 840)9000(10.714 \\
 340 & 840 \\
 \hline
 840)6120(7.286 & 6000 \\
 5880 & 5880 \\
 \hline
 2400 & 1200 \\
 1680 & 840 \\
 \hline
 7200 & 3600 \\
 6720 & 3360 \\
 \hline
 4800 & (240) \\
 4200 & \\
 \hline
 (600) & 
 \end{array}$$

Proof, A's 7 5 8  
B's 10 14 3

£. 18 0 0  
What

# PROPORTION.

112

What is the interest of 375 l. 12 s. 9½ d. for one year, at 4 per cent.?

l.	l.	l.
100	: 4 ::	375 12 9½
20		20
—		—
2000		7512
12		12
—		—
24000		90153
4		4
—		—
96000		360614
		4
		—
		l.
		96000)1442456(15.025=15 0 6
		96000
		—
		482456
		480000
		—
		245600
		192000
		—
		536000
		480000
		—
		(50000)

Or thus, more concisely :

l.
375 12 9 2 principal.
4 interest.
—
15,02 11 2 0
20
—
0,51
12
—
6,14
K 2

I laid out 150 l. upon tea at 7 s. 6 d. per lb.; but after it was damaged at sea, I sold it again for 125 l.; how much did I lose on each lb.?

$$\begin{array}{r} \text{£.} \quad \text{d.} \quad \text{l.} \\ 150 : 90 :: 125 \end{array}$$

$$\begin{array}{r} \text{---} \quad 12 \quad 7 \quad 6 \\ 150)11250(75=6 \quad 3 \\ \text{---} \quad 1050 \quad \text{---} \\ \text{---} \quad \quad \quad 1 \quad 3 \\ \quad \quad \quad 750 \\ \quad \quad \quad 750 \\ \text{---} \end{array}$$

Or thus, by di- 6) 7 6  
viding it by 6, ---  
because  $\frac{1}{6}$  of 1 3  
the whole was  
lost.

If I gain an halfpenny on a shilling, how much do I gain on 100 l.?

$$\begin{array}{r} \text{s.} \quad \text{f.} \quad \text{s.} \\ 1 : 2 :: 2000 \end{array}$$

$$\begin{array}{r} \text{---} \quad 2 \\ 4)4000 \\ \text{---} \\ 12)1000 \\ \text{---} \\ 2,0)8,3 \quad 4 \end{array}$$

$$\text{£. } 4 : 3 : 4$$

Or thus, 2 f. is  $\frac{1}{4}$  of 4) 100  
a shilling. ---

$$\begin{array}{r} 6)25 \\ \text{£. } 4 : 3 : 4 \end{array}$$

A bartered with B 24 dozen of stockings at 3 l. 6 s. 8 d. per dozen; and 244 yards linen at 3 s. 4 d. per yard; for which he wanted sugar at 3 l. 4 s. per Cwt. and rum at 9 s. per gallon; I demand how much he received of each, as he wanted of both an equal value?

24 dozen of stockings at 3 l. 6 s. 8 d. and 244 yards linen at 3 s. 4 d.

# PROPORTION:

113

$\frac{24}{3} = 8$   
 72  
 —  
 80  
 l. Cwt. l.  
 3.2 : 1 :: 60.333

l.  
 Stockings. 80  
 Linen. 40 13 4  
 —————  
 2) 120 13 4  
 —————  
 60 : 6 : 8

6) 244  
 —  
 40 : 13 : 4

Cwt. gr. lb. oz.  
 3.2) 60.333 (18.819 = 18 3 7 11  
 32 ——— 4  
 ——— s. g. l. s. d.  
 283 3.276 9 : 1 :: 60 6 8  
 256 28 12 20  
 ———  
 263 2208 108 206  
 256 552 12  
 ——— g. ch. g.  
 63 7.728 108) 14480 (134 1 1  
 32 16 108 rum.  
 ———  
 313 4368 368  
 288 728 324  
 (24) 11.648 440  
 432  
 ———  
 8  
 16 chopins.  
 ———  
 128 (1  
 108  
 ———  
 20  
 8  
 ———  
 160 (1  
 108  
 ———  
 (52)

There

There is a certain building raised in 8 months by 120 workmen ; but the same being demolished, it is required to be rebuilt in 2 months ; I demand how many men must be employed about it ?

$$\begin{array}{r}
 m. \quad m. \quad m. \\
 8 : 120 :: 2 \\
 \quad 8 \\
 \quad \hline
 2 \overline{) 960} \\
 \quad \hline
 480
 \end{array}$$

How many yards of matting, that is half-yard wide, will cover a room, that is 18 feet wide, and 30 feet long ?

$$\begin{array}{r}
 f. \quad y. \quad f. \\
 18 : 10 :: 1.5 \\
 \quad 10 \\
 \quad \hline
 1.5 \overline{) 180.0} (120 \text{ yards} \\
 \quad 15 \\
 \quad \hline
 \quad 30 \\
 \quad 15 \\
 \quad \hline
 \quad (0)
 \end{array}$$

A B merchant in Glasgow sends to his correspondent in Virginia 2456 yards of Osnaburgs at  
6 d.

# PROPORTION.

115

6 d. per yard, 456 yards of lawn at 4 s. per yard, and 145 dozen of napkins at 2 s. 6 d. each; for which he wants returns in tobacco at 18 l. per hogf-head, and cotton at 16 d. per lb.  $\frac{1}{3}$  value in cotton, and the rest in tobacco: *Quær.* how much did he receive of each?

245,6 Osnaburgs	Osnaburgs = 61 : 8
<u>          </u>	Lawns = 91 : 4
$\frac{1}{25} = 61 : 8$	Napkins = 217 : 10
5)456 Lawn	<u>          </u>
<u>          </u>	370 : 2
91 : 4	Cotton $\frac{1}{3} = 123 \quad 7 : 4$
145 dozen of napkins	<u>          </u>
72 : 10	Tobacco 246 : 14 : 8
<u>          </u>	
217 : 10	
d lb.      l. s. d.      l. hhd.      l. s d.	
16 : 1 : : 123   7   4      18 : 1 : : 246 14 8	
20                          20                          20	
<u>                                2467                          360                          4834</u>	
12                          12                          12	
<u>                                        16)29608(1850 <math>\frac{1}{2}</math>      4320                          )58016(13 <math>\frac{1}{2}</math></u>	
16    4320	
<u>                                        136    14816</u>	
128    12960	
<u>                                        80    (1856)</u>	
80	
<u>                                        8</u>	
16	

*Question*

*Question 1.* If 3 ounces of silver be worth 15 s. what is the price of 4 lb. at that rate? *Ans.* 12 l. Note, 12 oz. of silver equal 1 lb.

*Qu. 2.* If the interest of 100 l. for 1 year be 6 l. what is the interest of 530 l. for the same time? *Ans.* 31 l. 16 s.

*Qu. 3.* If 48 lb. of goods cost 8 s. what will be the price of 1 Cwt. at that rate? *Ans.* 11 l. 4 s.

*Qu. 4.* If 15 lb. of cochineal cost 24 l. what will be the price of 82 lb. at that rate?

*Ans.* 131 l. 4 s.

*Qu. 5.* What is the price of 1 piece of cloth, when 40 pieces cost 750 l. 15 s.?

*Ans.* 18 l. 15 s. 4½ d.

*Qu. 6.* If I buy cloth for 16 s. and sell it again for 18 s. per yard, how much do I gain upon every 100 l.? *Ans.* 12 l. 10 s.

*Qu. 7.* If a dozen ells of linen are worth 3 l. 6 s. how much will 8 pieces (each containing 54 ells) amount to? *Ans.* 118 l. 16 s.

*Qu. 8.* If 3 qrs. of velvet cost 7 s. 3 d. how many yards will be got for 13 l. 15 s. 6 d.?

*Ans.* 28½ yards.

*Qu. 9.* If I buy 2 Cwt. 1 qr. 7 lb. of coffee for 64 l. 15 s. at how much must I sell it per ounce to gain 21 l. 11 s. 8 d. on the whole? *Ans.* 5 d.

*Qu. 10.* What principal sum will yield 35 l. 15 s. interest, at 5 per cent. per annum?

*Ans.* 715 l.

*Qu. 11.* In what time will 100 l. principal yield 73 l. interest at 5 per cent. per annum?

*Ans.* 14 years 7 ⅔ months.

*Qu. 12.* If 150 pints of wine serve 15 men for 6 months, how many men will drink 1370 pints in the same time? *Ans.* 137 men.

*Qu. 13.* If a man performs a journey in 9 days, when the days is 11 hours, in what time will he perform the same, when the day is 15 hours?

*Ans.* 6 days 14 hours.

*Qu.*

*Qu.* 14. If the penny-loaf ought to weigh 9 ounces, when wheat is at 4 s. 6 d. per bushel, what ought it to weigh, when wheat is at 6 s. 9 d. per bushel? *Ans.* 6 ounces.

*Qu.* 15. At what price is the bushel of wheat, when the penny-loaf weighs 5 oz. 8 pwt. if it weighs 9 oz. when wheat is at 4 s. 6 d.? *Ans.* 7 s. 6 d. N. B. 20 pwt is 1 oz.

*Qu.* 16. If 42 s. worth of wine serve 12 men, when wine is sold at 25 l. 4 s. per hoghead, how many men will 42 s. worth serve when wine is sold at 18 l. 18 s. per hoghead? *Ans.* 16 men.

*Qu.* 17. A governor of a fort having provisions sufficient to serve 2820 men for 6 months, how many of them must be dismissed, that the provisions may serve for 2 months longer? *Ans.* 705 men.

*Qu.* 18. If I lend a man 650 l. for 22 months, how long ought he to lend me 953 l. 6 s. 8 d. to be even with me? *Ans.* 15 months.

*Qu.* 19. If 54 men build a wall in 94 days, what time will 36 men take to build the said wall? *Ans.* 141 days.

*Qu.* 20. If 14 l. worth of oats serve 57 horses, when oats are at 9 s. per boll, what will be the price of the boll of oats, when 45 horses eat 14 l. worth of oats? *Ans.* 11 s. 2½ d

*Qu.* 51. If 55 men can build a wall in 16 days, when they work 10 hours every day, how many hours must they work every day, that 40 men may build the same wall? *Ans.* 13¾ hours.

*Qu.* 22. I have by me 96 dozen of oranges, which cost me 4 l. 16 s. but they are somewhat damaged; so that I am willing to lose 24 s. on the whole: at what rate must I sell them per dozen? *Ans.* 9 d.

*Qu.* 23. A merchant sends to Spain 1300 pieces of broad cloth, each piece 47 yards, at 15 s. 6 d. per

per yard, in order to have returns from thence, the one half in wine, at 65 l. per tun, and the other half in oranges, at 3 l 10 s. per chest: what quantity of each will he get? *Ans.* 364 tuns 1 hogthead wine, and 6764 chests of oranges.

*Qu.* 24. If a merchant would mix 8 lb. of tea at 7 s. per lb. with 5 lb. d<sup>o</sup> at 10 s. and with 12 lb. d<sup>o</sup> at 14 s.; how can he sell 1 lb. of said mixture? *Ans.* 10 s. 11½ d.

*Nota,* As the sum of the given quantities of the several simples is to the total value, so is any quantity of the mixture to the price.

*Qu.* 25. Two merchants, A and B, barter goods; A hath 5 Cwt. 3 qrs. 14 lb. of pepper at 3 l. 10 s. per Cwt. and B hath cotton at 10 d. per lb. how much cotton must B give to A for his pepper? *Ans.* 493½ lb. cotton.

*Qu.* 26. A merchant bought 436 yards of cloth at 8 s. 6 d. per yard: and sold the same again at 10 s. 4 d. per yard, what did he gain thereon? *Ans.* 39 l. 19 s. 4 d.

*Qu.* 27. A grocer bought 3 hogsheds of sugar, each 10 Cwt. 3 qrs. 12 lb. gross, tare 26 lb. per hoghead at 2½ d. per lb. I demand what the 3 hogsheds came to? *Ans.* 37 l. 3 s. 9 d.

*Qu.* 28. A goldsmith sold a tankard for 10 l. 12 s. at the rate of 5 s. 4 d. per ounce: I demand the weight of it? *Ans.* 39 oz. 15 pwts.

### COMPOUND PROPORTION,

**F**ROM five numbers given finds a sixth, and is therefore called by some the rule of five. It is either direct, inverse, or mixt; and if mixt, it is either, first, direct and inverse, or, secondly, inverse and direct.

N. B.

N. B. Every compound question may be reduced into two simple ones. And in every compound question there are five numbers given, and three different species.

**RULE 1.** To state the question in the compound rule. Set that number which is of the same species with the number sought, in the third place; that which gives it, in the first; the number common to both, in the second; and that which is of the same species with the first number, in the fourth place; the fifth number sets itself.

**RULE 2.** To resolve any compound question, into its simple ones. The first, third, and fourth terms, make the first simple question. The second, the answer to the first, and fifth term, make the second simple question.

**RULE 3.** To know if a question is of the direct, inverse, or mixt rule; and if mixt, to know whether it is direct and inverse, or inverse and direct.

If both simple questions are direct, the compound question is direct. If both are inverse, the compound question is inverse.

If one simple question is direct, and the other inverse, the compound question is mixt.

If the first simple question is direct, and the second inverse, the compound mixt question is direct and inverse, and *e contra*.

**RULE 4.** To work the direct rule.

Multiply the two first terms for a divisor, and the other three for a dividend; the quotient gives the answer.

**RULE 5.** To work the inverse rule.

Multiply the two last terms for a divisor, and the three first for a dividend.

**RULE 6.** To work the direct and inverse rule.

Multiply the first and last terms for a divisor, and the middle three for a dividend.

**RULE**

# 120 COMPOUND PROPORTION.

**RULE 7.** To work the inverse and direct rule,  
Multiply the second and fourth terms for a divisor, and the remaining three for a dividend.

**N. B.** It will not be improper for all the questions in this rule to be wrought both ways, *i. e.* by one operation, and also by two simple operations, because they will not only improve the scholar, but likewise reciprocally prove each other; although the proof of direct proportion is naturally the product of the two extremes equal to the product of the two means. And in inverse proportion the product of the two first terms is equal to the product of the last, and the answer.

## EXAMPLES.

If 5 l. is the interest of 100 l. for 365 days, what is the interest of 798 l. for 86 days?

$$\begin{array}{cccccc} pr. & d. & l. & pr. & d. & \\ 100 & : & 365 & : & 5 & : : 798 & : & 86 \end{array}$$

$$\begin{array}{r} 100 \\ \hline 36500 \end{array} \quad \begin{array}{r} 5 \\ \hline 3990 \\ 86 \end{array}$$

$$\begin{array}{r} 23940 \\ 31920 \\ \hline \end{array}$$

$$\begin{array}{r} 36500 \quad 343140 \quad l. s. \\ 328500 \end{array}$$

$$\begin{array}{r} 146400 \\ 146000 \\ \hline \end{array}$$

$$(400)$$

# COMPOUND PROPORTION. 121

$$\begin{array}{r} l. \quad l. \quad l. \\ 100 : 5 :: 798 \\ \quad \quad 5 \\ \hline \end{array}$$

$$1,00) 39,90$$

$$\begin{array}{r} d. \quad l. \quad d. \\ 365 : 3990 :: 86 \\ \quad \quad 86 \\ \hline \end{array}$$

$$\begin{array}{r} 23940 \\ 31920 \\ \hline \end{array}$$

$$\begin{array}{r} l. \quad s. \\ 365) 343140 \quad (9.4=9 \quad 8 \\ 3285 \\ \hline \end{array}$$

$$\begin{array}{r} 1464 \\ 1460 \\ \hline \end{array}$$

(4)

It 12 inches of length, and 12 of breadth, require 12 of thickness, to make a solid foot, what will 6 of length, and 4 of breadth, require?

$$\begin{array}{r} l. \quad b. \quad th. \quad l. \quad b. \quad l. \quad th. \quad l. \quad b. \quad th. \quad b. \\ 12 : 12 : 12 :: 6 : 4 \quad 12 : 12 :: 6 \quad 12 : 24 :: 4 \end{array}$$

$$\begin{array}{r} 12 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \hline \end{array}$$

$$\begin{array}{r} 12 \\ \hline \end{array}$$

$$\begin{array}{r} 12 \\ \hline \end{array}$$

$$\begin{array}{r} 144 \\ \hline \end{array}$$

$$\begin{array}{r} 24 \\ \hline \end{array}$$

$$6) 144 (24$$

$$4) 288 (72$$

$$\begin{array}{r} 12 \\ \hline \end{array}$$

$$\begin{array}{r} 12 \\ \hline \end{array}$$

$$\begin{array}{r} 28 \\ \hline \end{array}$$

$$24) 1728 (72 \text{ inches.}$$

$$\begin{array}{r} 168 \\ \hline \end{array}$$

$$\begin{array}{r} 24 \\ \hline \end{array}$$

$$\begin{array}{r} 24 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \hline \end{array}$$

$$\begin{array}{r} 48 \\ \hline \end{array}$$

$$\begin{array}{r} 48 \\ \hline \end{array}$$

If 18 roods of ditching be done by 3 men in 6 days, how many roods will be wrought by 8 men in 4 days, at the same rate?

† L

m.

# 122 COMPOUND PROPORTION.

<i>m.</i>	<i>d.</i>	<i>r.</i>	<i>m.</i>	<i>d.</i>	<i>m.</i>	<i>r.</i>	<i>m.</i>	<i>d.</i>	<i>r.</i>	<i>d.</i>				
3	6	18	:	8	3	18	:	8	6	48				
6			:	4	8			:	4					
<hr/>					<hr/>					<hr/>				
18					3	)	144			6	)	192		
<hr/>					<hr/>					<hr/>				
							48					32 roods.		
<hr/>					<hr/>					<hr/>				

For to multiply by 18, and divide by 18, leaves the number as it was.

If 12 men work 96 roods of work in 15 days, in what time will 8 men work 120 roods, at that rate of working?

<i>m.</i>	<i>r.</i>	<i>d.</i>	<i>m.</i>	<i>r.</i>			
12	96	:	15	:	8	:	120
	8			15			
<hr/>				<hr/>			
768				1800			
				12			
				<i>d. h.</i>			
				768	)	21600	(28 3
				<hr/>			
				1536			
				<hr/>			
				6240			
				6144			
				<hr/>			
				96			
				24			
				<hr/>			
				384			
				192			
				<hr/>			
				2304	(3		
				2304			
				<hr/>			

# COMPOUND PROPORTION. 123

<i>m.</i>	<i>d. m.</i>	<i>r. d.</i>	<i>h.</i>
12	15	8	96 : 22
12			12 : 120
	<i>d. h.</i>		
8	180	(22 12	540
	16		120
			24 <i>d. h.</i>
20	96	64800	(675 (28 3
16		576	48
4		720	195
24		672	192
96		480	(3)
8		480	
16			
16			

A and B put in stocks for trade ; A puts in 48 l. and at 4 months end takes out 20 l. but 2 months after puts in 50 l. B puts in 60 l. and at the end of 4 months puts in 25 l. more ; at the end of 8 months they have gained 38 l. : what is each man's share of the gain ?

$$\begin{array}{r} \text{A. } 48 - 20 = 28 + 50 = 78 \\ \underline{4} \quad \quad \underline{2} \quad \quad \underline{2} \\ 192 \quad \quad 56 \quad \quad 156 = 404 \end{array}$$

$$\begin{array}{r} \text{B } 60 + 25 = 85 \\ \underline{4} \quad \quad \underline{4} \\ 240 \quad \quad 340 = \quad \quad 580 \\ \underline{\quad} \quad \underline{\quad} \quad \quad \underline{\quad} \\ \quad \quad \quad 984 \end{array}$$

# 124 COMPOUND PROPORTION.

$$984 : 38 :: 580$$

£.

$$984) 22040 \quad (22.398 = 22 : 7 : 11\frac{1}{2})$$

1968

2360

1968

3920

2952

9680

8856

8240

7872

(368)

$$984 : 38 :: 404$$

38

3232

1212

£.

$$984) 15352 \quad (15.601 = 15 : 12 : 0\frac{1}{2})$$

984

5512

4920

5920

5994

Proof A 15 12 0 $\frac{1}{2}$

B 22 7 11 $\frac{1}{2}$

38 0 0

1600

984

(616)

Four

Four merchants in company have bought a ship for 4840 l. whereof the first paid  $\frac{1}{2}$ , the second  $\frac{1}{3}$ , the third  $\frac{1}{5}$ , and the fourth  $\frac{1}{6}$ ; I demand what each merchant paid for his share of the ship?

$$\frac{1}{2} \frac{1}{3} \frac{1}{5} \frac{1}{6} = \frac{60}{180} \frac{90}{180} \frac{36}{180} \frac{30}{180} = \frac{216}{180}$$

$216 : 4840 :: 60$ $216 : 4840 :: 90$ $216 : 4840 :: 36$ $216 : 4840 :: 30$	}	pays	$\text{£. } 2016 \text{ } 13 \text{ } 4$ $1344 \text{ } 8 \text{ } 10\frac{2}{3}$ $806 \text{ } 13 \text{ } 4$ $672 \text{ } 4 \text{ } 5\frac{1}{2}$
--	---	------	--

Proof 4840 : 0 : 0

Three merchants in company have gained 498 l. which is to be divided among them after this manner, *i. e.* as the first hath  $\frac{2}{3}$ , the second shall have  $\frac{1}{3}$ ; and as the second hath  $\frac{1}{3}$ , the third shall have  $\frac{1}{3}$ : *Quar.* what each merchant received of said gain?

$$\frac{2}{3} \frac{1}{3} \frac{1}{3} = \frac{20}{30} \frac{10}{30} \frac{10}{30} = \frac{200}{300}$$

$2880 : 498 :: 768$ $2880 : 498 :: 1152$ $2880 : 498 :: 960$	}	gained	$\text{£. } 132 \text{ } 16$ $199 \text{ } 4$ $166 \text{ } 0$
--	---	--------	--

Proof 498 0

What is the interest of 507 l. 13 s. 7 d. at 5 per cent. for 4 years?

L 3

L

# 126 COMPOUND PROPORTION.

£. y. l. l. s. d. y.  
100 : 1 : 4 :: 507 13 7 : 4

1  
100  
10153  
12

121843  
5

609215  
4

1,00)24368,60X4=240÷100=2

$\frac{1}{11} = 2030 \ 8$

$\frac{2}{10} \ 101 \ 10 \ 8 \ 2$

507 13 7  
4

2030 14 4  
5

101.53 11 8  
20

10.71  
12

8.60  
4

240

£.  
Or thus, 5)507 13 7

101 : 10 : 8 : 2

N. B.

# COMPOUND PROPORTION. 127

N. B. When the rate of interest being multiplied by the time makes up precisely any aliquot part of 100, you need divide only the principal by the said aliquot part; and we shall subjoin here the aliquot parts of 100.

$10 = \frac{1}{10}$ ,  $20 = \frac{1}{5}$ ,  $25 = \frac{1}{4}$ ,  $30 = \frac{1}{3}$ ,  $40 = \frac{2}{5}$ ,  $50 = \frac{1}{2}$ ,  
 $60 = \frac{2}{3}$ ,  $70 = \frac{7}{10}$ ,  $75 = \frac{3}{4}$ ,  $80 = \frac{4}{5}$ , and  $90 = \frac{9}{10}$ .

What is the interest of 864 l. 16 s. 8 d. for 15 years, at 5 *per cent.*?

$$15 \times 5 = 75 = \frac{1}{4} \quad 4) \begin{array}{r} 864 \ 16 \ 8 \\ \underline{216 \ 4 \ 2} \\ \phantom{0} 3 \\ \underline{\phantom{0} 648 \ 12 \ 6} \end{array} \quad \text{Or thus, } \begin{array}{r} 864 \ 16 \ 8 \\ \phantom{000} 3 \\ \hline 4) 2594 \ 10 \ 0 \\ \underline{\phantom{000} 648 \ 12 \ 6} \end{array}$$

What is the interest of 334 l. from the 27th of July 1750, to the 24th of February 1769, excluding 11 days for the alteration of the style, at 5 *per cent.*?

$$\begin{array}{cccc} l. & s. & l. & y. & d. \\ 100 : 365 : 5 :: 334 : 18 : 201 \end{array} \quad \text{Ans. } 309 \text{ l. } 15 : 11.$$

What is the interest of 328 l. 14 s. 10 d. from the 28th of July 1764 to this day, being the 15th of March 1770, interest at  $4\frac{1}{2}$  *per cent.*?

328

# 128 COMPOUND PROPORTION.

$$\begin{array}{r}
 328 \ 14 \ 10 \\
 \hline
 4\frac{1}{4} \\
 \hline
 1314 \ 19 \ 4 \\
 82 \ 3 \ 8\frac{1}{2} \\
 \hline
 13,97 \ 3 \ \frac{1}{2} \\
 20 \\
 \hline
 19,43 \\
 12 \\
 \hline
 5,16
 \end{array}
 \qquad
 \begin{array}{l}
 \text{Ans. } 13 \ 19 \ 5 \text{ for 1 year.} \\
 \hline
 5 \\
 \hline
 69 \ 17 \ 1 \text{ for 5 years.} \\
 8 \ 16 \ 10 \text{ for 231 days.} \\
 \hline
 \text{£. } 78 \ 13 \ 11 \\
 \hline
 \hline
 \end{array}$$

$\text{If } 365 : 13 \ 19 \ 5 :: 231 \text{ } \text{Ans. } \overset{\text{£.}}{8 \ 16 \ 10}$

An usurer put out 455 l. principal at interest ; and after it had continued 3 years and 4 months, he received, for principal and interest together, 576 l. 6s, 8 d. at what rate *per cent. per annum* did he receive interest ?

$$\begin{array}{r}
 \text{£. } 576 : 6 : 8 \text{ Prin. and Int.} \\
 455 \text{ Prin. lent out,} \\
 \hline
 121 \ 6 \ 8 \\
 20 \\
 \hline
 2426 \\
 12 \\
 \hline
 29120
 \end{array}$$

£.

# COMPOUND PROPORTION. 129

£.	m.	d.	l.	m.
455 : 40 : 29120 :: 100 : 12				
40		12		
-----		-----		
18200		349440		
		100		
		-----		
		12		
182,00		349440,00		
		182		
		-----		
		2,0		
		16,0		
		-----		
		1674		
		1638		
		-----		
		364		
		364		
		-----		
		(0)		

## EXAMPLES for PRACTICE.

*Question 1.* If the carriage of 20 packs cost 16 l. for 136 miles, what will the carriage of 12 packs be for 28 miles? *Ans.* 1 l. 19 s. 6½ d.

*Qu. 2.* If 60 Cwt. cost 14 l. 10 s. for being carried 20 miles, what will 15 Cwt. cost to be carried 30 miles, at that rate? *Ans.* 5 l. 8 s. 9 d.

*Qu. 3.* If 48 s. be wages for 6 men for 6 days, how much wages must 48 men have for 30 days, at that rate? *Ans.* 96 l.

*Qu. 4.* If 10 cannons spend 40 barrels of powder in 1 day, how many barrels of powder will 24 cannons spend in 30 days? *Ans.* 2880.

*Qu. 5.* If 100 l. yield 5 l. interest in 1 year, how much interest will 850 l. yield in 3 years and 8 months? *Ans.* 155 l. 16 s. 8 d.

*Qu. 6.* What principal will yield 155 l. 16 s. 8 d. interest

# 130 COMPOUND PROPORTION.

interest in 3 years and 8 months, at the rate of 5 per cent. *per annum*? *Ans.* 850 l.

*Qu.* 7. In what time will 850 l. principal yield 155 l. 16 s. 8 d. interest, at the rate of 5 per cent. *per annum*? *Ans.* 3 years 8 months.

*Qu.* 8. On the 25th day of March 1764, a certain person lent on a mortgage 760 l. and the 25th day of May 1770, he received for interest thereof 281 l. 4 s. I demand at what rate *per cent. per annum* his money was lent? *Ans.* at 6 per cent.

*Qu.* 9. If the carriage of 60 Cwt. for 20 miles cost 14 l. 10 s. what weight ought to be carried 30 miles for 5 l. 8 s. 9 d.? *Ans.* 15 Cwt.

*Qu.* 10. If 60 Cwt. be carried 20 miles for 14 l. 10 s. how many miles ought 15 Cwt. to be carried for 5 l. 8 s. 9 d.? *Ans.* 30 miles.

*Qu.* 11. If 8 men mow 112 acres of grass in 14 days, how many men can mow 2000 acres at that rate in 10 days? *Ans.* 200 men.

*Qu.* 12. If 2000 acres of grass are mown by 200 men in 10 days, how many acres will 8 men mow in 14 days? *Ans.* 112 acres.

*Qu.* 13. If 170 bushels of wheat serve 680 men for 6 days, how much will serve 79200 men for 16 days? *Ans.* 6600 quarters.

*Qu.* 14. If 275 men cast a trench of 250 roods in 12 hours, how many men at that rate will cast a trench of 880 roods in 8 hours? *Ans.* 1452.

*Qu.* 15. If a wall of 12 feet long and 12 feet breadth, is also to be made 12 feet high, what will be the height of a wall, of 4 feet long, and 6 feet broad; so that it and the former may be of equal contents? *Ans.* 72.

*Qu.* 16. If 10 bushels of oats serve 18 horses for 20 days, how many bushels will serve 60 horses for 36 days? *Ans.* 60 bushels.

*Qu.* 17. If a vessel holding 700 gallons of liquor, be 3 feet deep, 5 feet long, and 4 feet broad,

broad, what must the breadth of a vessel be, to hold 700 gallons, when it is to be 5 feet deep and 6 feet long? *Ans.* 2 feet broad.

*Qu.* 18. Three traders, A, B, and C, have dealt in company; A put into the common stock, upon the 1st of January 1000 l. B, upon the 5th of January, put in 500 l. and C, upon the 12th of July, put in 800 l. at the year's end they balance accounts, and find 180 l. gained. *Quer.* each partner's share of said gain? *Ans.* A's 96 l. 1 s. 4 d. B's 47 l. 10 s. 1½ d. C's 36 l. 8 s. 6½ d.

*Qu.* 19. Three persons, A, B, and C, were partners in trade; A deposited 1200 l. upon the 1st of January, and upon the 1st of April he withdrew 200 l.; B put in 600 l. upon the 1st of March, and upon the 1st of August he added 250 l. more; C put in 500 l. upon the 1st of July, and on the 1st of October he took out 100 l. At the year's end they balanced accounts, and find they had gained 300 l. what must each partner's share of said gain be?

*Ans.* A's 167 l. 12 s. 6½ d. B's 96 l. 9 s. 0¼ d. and C's 35 l. 18 s. 5 d.

*Qu.* 20. Three merchants, A, B. and C, join their stocks together, and make up 12400 l. with which they traded, and gained 2480 l. of which A gets 686 l. B 870 l. and C 924 l. I demand what each partner put in? *Ans.* A 3430 l. B 4350 l. and C 4620 l.

*Qu.* 21. A footman travels 240 miles in 12 days, when the day is 12 hours long; how many days will he take to travel 720 miles, when the day is 16 hours long? *Ans.* 27 days.

*Qu.* 22. What is the interest of 200 l. for 3 years and ¾, at 5 per cent. per annum?

*Ans.* 37 l. 10 s.

Compound proportion may extend to 7, 9, or 11 given numbers, and an 8th, 10th, or 12th, number sought.

The

## 132 COMPOUND PROPORTION.

The method of stating the question depends upon the same principles, and the manner of proceeding is the same, as in the rule of five.

N. B. Every compound question may be resolved into as many simple questions as there are terms on the left hand of the middle term.

RULE. To work any compound question.

Having resolved your compound question into all its simple ones, so as the middle term of the compound question shall be the 3d term in every simple question, multiply all the antecedents of the first ratios continually for a new antecedent, and all their consequents for the consequent of a new ratio; and say, as this new antecedent is to its consequent, so is the common third term to a fourth proportional, which is the answer of the question.

### EXAMPLES.

If 15 men eat 13 s. worth of bread in 6 days, when wheat is at 12 s. per boll, what will be the price of bread to serve 30 men for 12 days, at the same rate of eating, when wheat is at 10 s. per boll?

*m. d. s. s. m. d. s.*

15 : 6 : 12 : 13 :: 30 : 12 : 10 which when resolved to three simple questions stands thus.

$$15 : 30 :: 13 \text{ 4th}$$

$$6 : 12$$

$$12 : 10$$

And  $15 \times 6 \times 12 : 30 \times 12 \times 10 :: 13 : \text{the Ans. 2 l. 3 s. 4 d.}$

If

# COMPOUND PROPORTION. 133

If 18 roods of ditching be wrought by 3 men in 16 days, when the day is 15 hours long; how much will be done by 8 men in 4 days, when they work 9 hours a-day? *Ans.*  $7\frac{1}{2}$  roods.

If 6 yards of linen be worth 12 yards of drugget, or 4 yards of cloth, or 30 yards of muslin, or 60 lb. of tobacco; how many yards of linen will be worth 16 yards of drugget, 20 yards of cloth, 9 yards of muslin, or 360 lb. of tobacco?

*d. cl. m. t. l. d. cl. m. t.*

12 : 4 : 30 : 60 : 6 :: 16 : 20 : 9 : 360

12 : 16 :: 6 4th

4 : 20

30 : 9

60 : 360

And  $12 \times 4 \times 30 \times 60 = 86400$ . Again,  $16 \times 20 \times 9 \times 360 = 1036800$

$86400 : 6 :: 1036800$

6

$864,00)62208,00(72$  yards of linen.

6048

1728

1728

If 12 men build a wall 30 feet long, 6 high, and 3 thick, in 15 days, when the day is 12 hours; in how many days will 60 men build a wall 300 feet long, 8 high, and 6 thick, when they work 8 hours each day?

*m. l. h. th. h. d. m. l. h. th. h.*

12 : 30 : 6 : 3 : 12 : 15 :: 60 : 300 : 8 : 6 : 8

60 : 12 :: 15 4th

30 : 300

6 : 8

3 : 6

8 : 12

† M

And

And  $60 \times 30 \times 6 \times 3 \times 8 = 259200$  divisor; again,  $12 \times 300 \times 8 \times 6 \times 12 = 2073600 \times 15 = 31104000 \div 259200 = 120$  days.

If 100 lb. of Venice weigh 70 lb. of Lyons, and 120 lb. of Lyons weigh 100 lb. of Roan, and 80 lb. of Roan weigh 100 lb. of Toulouse, and 100 lb. of Toulouse 74 lb. of Geneva; how many pounds of Geneva will 100 lb. of Venice weigh?  
*Ans.*  $53\frac{2}{3}$  lb.

## A D D I T I O N of V U L G A R F R A C - T I O N S.

**W**E have on purpose disjoined Reduction and Addition, &c. of Vulgar Fractions, for this obvious reason, that the learner may be obliged to revise Reduction, after the intervention of Practice and Proportion; because this second review of it will make a deeper and more lasting impression upon his mind, and consequently render him more alert in the following operations.

As things of the same species and kind can only be added or subtracted, &c. therefore, before we begin to add or subtract, &c. fractions, you will please

Observe, 1. If you have integers mixt with your fractions, first of all reduce these integers to the form of a fraction; thus,  $8 - \frac{2}{3} = \frac{8}{1} - \frac{2}{3}$ .

Observe, 2. If you have mixt numbers, reduce them to improper fractions; thus,  $8\frac{1}{2} + \frac{1}{3} = \frac{17}{2} + \frac{1}{3}$ .

Observe, 3. If you have compound fractions, reduce them to simple ones; thus,  $\frac{8}{9} + \frac{1}{4}$  of  $\frac{6}{8} = \frac{8}{9} + \frac{1}{4}$ .

**R U L E.** Reduce your fractions to a common denominator,

denominator, (if they are not already so), and add the numerators only, continuing your common denominator.

# EXAMPLES.

1.  $\frac{2}{3} + \frac{1}{3} = \frac{3}{3}$

2.  $\frac{1}{4} + \frac{1}{8} = \frac{2}{8} + \frac{1}{8} = \frac{3}{8}$

3.  $6\frac{1}{3} + \frac{4}{3} = \frac{20}{3} + \frac{4}{3} = \frac{24}{3} = 8$

4.  $\frac{7}{9} + \frac{2}{3}$  of  $\frac{4}{5} = \frac{7}{9} + \frac{8}{9} = \frac{15}{9} = \frac{5}{3}$

5.  $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{4}{8} + \frac{2}{8} + \frac{1}{8} = \frac{7}{8}$

Or thus,  $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{4}{8} + \frac{2}{8} + \frac{1}{8} = \frac{7}{8}$

Add  $\frac{7}{8}$  of a pound to  $\frac{1}{4}$  of a shilling. *Ans.* 18 s. 3 d.

Add  $\frac{1}{4}$  of a penny to  $\frac{1}{8}$  of a pound. *Ans.* 2 s. 3 d.  $1\frac{6}{8}$  f.

Add  $\frac{1}{2}$  of a pound Troy to  $\frac{7}{8}$  of an oz. *Ans.* 6 oz. 11 pwt. 16 gr.

Add  $\frac{1}{2}$  of a yard to  $\frac{2}{3}$  of a foot. *Ans.* 2 feet 2 inches.

Add  $\frac{1}{3}$  of a day to  $\frac{1}{2}$  of an hour. *Ans.* 8 hours 30 minutes.

Add  $\frac{2}{3}$  of a yard,  $\frac{1}{4}$  of a foot, and  $\frac{1}{8}$  of a mile. *Ans.* 1540 yards 2 f. 9 inches.

# 186 SUBTRACTION of

## SUBTRACTION of VULGAR FRACTIONS.

**RULE.** **R**educer your fractions to a common denominator, (if they are not so already), and subtract the numerators only, continuing your common denominator.

### EXAMPLES.

$$1. \frac{8}{15} - \frac{3}{15} = \frac{5}{15} = \frac{1}{3}$$

$$2. \frac{8}{9} - \frac{2}{3} = \frac{8}{9} - \frac{6}{9} = \frac{2}{9}$$

$$3. 8 - \frac{2}{3} = \frac{24}{3} - \frac{2}{3} = \frac{22}{3} = 7\frac{2}{3}$$

$$4. 4\frac{2}{3} - \frac{1}{3} = 4\frac{4}{3} - \frac{1}{3} = 4\frac{3}{3} = 4$$

$$5. \frac{8}{9} - \frac{2}{3} \text{ of } \frac{5}{6} = \frac{8}{9} - \frac{10}{9} = -\frac{2}{9}$$

From  $\frac{1}{2}$  of a pound take  $\frac{1}{4}$  of a shilling. *Ans.* 9 s. 3 d.

From  $\frac{1}{2}$  of a shilling take  $\frac{1}{4}$  of a penny. *Ans.* 5<sup>1</sup> d.

From  $\frac{1}{2}$  of an ounce take  $\frac{1}{4}$  of a pwt. *Ans.* 11 pwts 3 gr.

From  $\frac{1}{2}$  of an Cwt. take  $\frac{1}{4}$  of a pound. *Ans.* 1 qr. 27 lb. 6 oz. 10<sup>3</sup>/<sub>4</sub> dr.

From 7 weeks take 9<sup>7</sup>/<sub>8</sub> days. *Ans.* 5 w. 4 d. 7 hours 12 m.

From 4 days 7<sup>1</sup>/<sub>2</sub> h. take 1 day 9<sup>1</sup>/<sub>8</sub> hours. *Ans.* 2 days 22<sup>1</sup>/<sub>8</sub> hours.

MUL-

## MULTIPLICATION of VULGAR FRACTIONS.

**M**ultiply the numerators for the numerator of the product, and the denominators for its denominator.

### EXAMPLES.

$$1. \frac{3}{4} \times \frac{4}{5} = \frac{3}{5}$$

$$2. \frac{1}{2} \times 8 = \frac{1}{2} \times \frac{8}{1} = \frac{8}{2} = 4$$

$$3. \frac{1}{4} \times 6\frac{1}{2} = \frac{1}{4} \times \frac{13}{2} = \frac{13}{8} = 1\frac{5}{8}$$

$$4. \frac{7}{8} \times \frac{3}{4} \text{ of } \frac{4}{5} = \frac{7}{8} \times \frac{3}{5} = \frac{21}{40}$$

Multiply  $7\frac{1}{2}$  by  $8\frac{1}{2}$ . *Ans.*  $61\frac{1}{4}$ .

Multiply  $\frac{1}{2}$  of 7 by  $\frac{1}{8}$ . *Ans.*  $1\frac{1}{4}$ .

Multiply  $\frac{1}{8}$  by  $\frac{4}{5}$  of 11. *Ans.*  $2\frac{1}{5}$ .

## DIVISION of VULGAR FRACTIONS.

**M**ultiply cross-ways, viz. the numerator of the dividend, by the denominator of the divisor, for the numerator of the quot, &c.

### EXAMPLES.

$$1. \frac{2}{3} \div \frac{1}{5} = \frac{2}{3} \times \frac{5}{1} = \frac{10}{3} = 3\frac{1}{3}$$

$$2. \frac{3}{4} \div 8 = \frac{3}{4} \times \frac{1}{8} = \frac{3}{32}$$

# 138 PROPORTION of

$$3. 6) \frac{4}{5} (= \frac{6}{5}) \frac{4}{5} (\frac{4}{5} = \frac{2}{5})$$

$$4. \frac{3}{4} \cdot 2 \frac{2}{3} (= \frac{3}{4}) \frac{1}{2} (\frac{4}{3} = 3 \frac{1}{3} = 3 \frac{2}{3})$$

$$5. \frac{5}{6} \cdot \frac{1}{2} \text{ of } \frac{4}{5} (= \frac{5}{6}) \frac{3}{5} (\frac{4}{5} = \frac{2}{5})$$

Divide  $\frac{7}{8}$  by 4. *Ans.*  $\frac{7}{32}$ .

Divide 99 by 108. *Ans.*  $\frac{11}{12}$ .

Divide  $4 \frac{1}{2}$  by  $\frac{2}{3}$  of 4. *Ans.*  $2 \frac{1}{2}$ .

## PROPORTION of VULGAR FRACTIONS.

**P**roportion of Vulgar Fractions is stated and wrought precisely by the same rules as Proportion of integers; only care must be taken to reduce such fractions as require it, the practice whereof will be rendered very easy by the operation of the following

### EXAMPLES.

If  $\frac{3}{4}$  of a yard of silk cost 3 s. 4 d. what will 27 yards cost?

$$\begin{array}{rcl} y. & l. & y. \\ \frac{3}{4} : \frac{1}{6} :: \frac{1}{2} : \frac{1}{2} \end{array} \quad \frac{1}{2} \cdot 27 = 13 \frac{1}{2} = 13 \text{ l. } 6 \text{ s. } 6 \text{ d.}$$

What cost 16 yards of broad cloth, at 2 s. 6 d. per quarter?

$$\begin{array}{rcl} y. & l. & \\ \frac{1}{4} : \frac{1}{8} :: \frac{1}{2} : \frac{1}{2} \end{array} \quad \frac{1}{2} \cdot 16 = 8 \text{ l.}$$

If

If  $\frac{5}{8}$  of a yard of velvet cost 15 s. what will 9 yards give?

$$\begin{array}{rcl} y. & l. & y. \\ \frac{5}{8} : \frac{1}{2} :: \frac{2}{1} = \frac{1}{4} & \frac{1}{2} \times \frac{1}{4} = \frac{1}{8} & \frac{1}{8} \times 15 = 10 \frac{1}{2} \end{array}$$

When  $8\frac{1}{2}$  yards of holland cost 2 l. 2 s. 6 d. what will  $\frac{1}{4}$  of a yard cost?

$$\begin{array}{rcl} y. & l. & y. \\ \frac{17}{2} : \frac{1}{4} :: \frac{1}{4} = \frac{1}{4} & \frac{1}{4} \times \frac{17}{2} = \frac{17}{8} & \frac{17}{8} \times 2 \text{ l. } 2 \text{ s. } 6 \text{ d.} = 1 \text{ l. } 3 \text{ s. } 9 \text{ d.} \end{array}$$

If  $\frac{1}{2}$  yard of cloth cost 3 s. 9 d. what will 15 yards cost?

$$\begin{array}{rcl} y. & l. & y. \\ \frac{1}{2} : \frac{1}{40} :: \frac{1}{2} = \frac{1}{2} & \frac{1}{2} \times \frac{1}{40} = \frac{1}{80} & \frac{1}{80} \times 3 \text{ s. } 9 \text{ d.} = 15 \text{ s. } 12 \text{ d. } 6 \end{array}$$

If  $\frac{3}{4}$  of a lb. of sugar cost 4 d. what will 1 Cwt. cost?

$$\begin{array}{rcl} lb. & l. & lb. \\ \frac{3}{4} : \frac{1}{40} :: \frac{1}{4} = \frac{1}{4} & \frac{1}{4} \times \frac{1}{40} = \frac{1}{160} & \frac{1}{160} \times 4 \text{ d.} = 2 \text{ s. } 9 \text{ d. } 9 \frac{1}{2} \end{array}$$

If 1 Cwt. of sugar cost 50 s. what will  $\frac{1}{4}$  of a lb. cost?

$$\begin{array}{rcl} lb. & l. & lb. \\ \frac{1}{4} : \frac{1}{40} :: \frac{1}{4} = \frac{1}{4} & \frac{1}{4} \times \frac{1}{40} = \frac{1}{160} & \frac{1}{160} \times 50 \text{ s.} = 1 \text{ l. } 1 \text{ s. } 5 \text{ d.} \end{array}$$

If 25 men do a piece of work in 8 weeks and 4 days, how many men will do the same piece of work in 3 weeks and 3 days?

$$\begin{array}{rcl} w. & m. & w. \\ 8\frac{1}{2} : 25 :: 3\frac{1}{2} = \frac{7}{2} & \frac{7}{2} : \frac{25}{7} = \frac{25}{7} & \frac{25}{7} \times \frac{7}{2} = 62\frac{1}{2} \end{array}$$

If 72 Cwt. 2 qrs. 4 lb. cost 157 l. 4 s. what will  
168 Cwt. 8 lb. cost ?

$$\begin{array}{rcl} \text{Cwt.} & \text{l.} & \text{Cwt.} \quad \text{l. s. d.} \\ 72 \frac{60}{112} : 157 \frac{4}{10} :: 168 \frac{8}{112} = \frac{8124}{112} : \frac{3814}{10} :: \frac{28824}{112} = \end{array}$$

$$\frac{8124}{112} \times \frac{9182656}{2240} (= \frac{6628457462}{112000}) = 364 \text{ } 4 \text{ } 11.$$

## A D D I T I O N and S U B T R A C. T I O N of D E C I M A L S.

**RULE.** **W**ORK as in integers, but remember to place all your decimal points precisely below each other.

### E X A M P L E S of A D D I T I O N.

.75	86.5
.895	79.725
.5	18.75
.625	24.5
.78	92.865
.125	<hr/>
<hr/>	306.34
3.675	<hr/>

Miles.	Lb.
41.8102	3.18104
140.037	1.14
18.10	7.181
7.8141	8.7121
16.4612	13.19817
7.81	86.071
<hr/>	<hr/>

**E X.**

# MULTIPLICATION of DECIMALS. 141

## EXAMPLES of SUBTRACTION.

$$\begin{array}{r} .75 \\ .275 \\ \hline .475 \end{array}$$

$$\begin{array}{r} 82.125 \\ 26.75 \\ \hline 55.375 \end{array}$$

	<i>Years</i>
From	1081.761
Take	<u>10.00012</u>

<i>Weeks</i>
127.19
<u>121.</u>

	<i>Months</i>
From	6100.
Take	<u>6.109</u>

<i>Ells</i>
.172618
<u>.0000148</u>

## MULTIPLICATION of DECIMALS.

**R**ULE. Work as in integers, and give as many decimal places to the product as are in both factors; but if it consists not of so many places, you must supply that defect by adding one or more ciphers.

E X.

## EXAMPLES.

Mult. .785  
By .75

---

3925  
5495  

---

.58875

76.85  
9.5

---

38425  
69165  

---

730.075

.325  
.25

---

625  
250  

---

.03125

.8672  
.0054

---

34688  
43360  

---

.00468288

## DIVISION of DECIMALS.

**R**ULE. Work as in integers, and allow as many decimal places to the quot, as the dividend has more than the divisor.

EXAMPLES.

$$3.75 \overline{) 87.94345} (23.45$$

750

1294

1125

1693

1500

1934

1875

595

375

220

$$32.5 \overline{) 76.75} (23$$

650

1175

975

200

Or

# 144 DIVISION of DECIMALS.

Or it may be continued to a decimal, and stands thus :

$$\begin{array}{r}
 3.25 \overline{) 76.75(23.615} \\
 \underline{650} \phantom{00} \\
 1175 \phantom{00} \\
 \underline{975} \phantom{00} \\
 2000 \phantom{00} \\
 \underline{1950} \phantom{00} \\
 500 \phantom{00} \\
 \underline{325} \phantom{00} \\
 1750 \phantom{00} \\
 \underline{1625} \phantom{00} \\
 125 \phantom{00} \\
 \underline{\phantom{00}}
 \end{array}$$

N. B. If your divisor has decimal places, and your dividend has none, or if your divisor has more decimal places than the dividend, supply the deficient places in your dividend with ciphers, and the quot is an integer, as in the following

EX.

EXAMPLES.

$  \begin{array}{r}  3.25)7968( \\  3.25)7968.00(2451 \\  \underline{650} \\  1468 \\  \underline{1300} \\  1680 \\  \underline{1625} \\  550 \\  \underline{325} \\  225 \\  \underline{\phantom{0}}  \end{array}  $	$  \begin{array}{r}  2.375)7658.5( \\  2.375)7658.500(3224 \\  \underline{7125} \\  5335 \\  \underline{4750} \\  5850 \\  \underline{4750} \\  11000 \\  \underline{9500} \\  1500 \\  \underline{\phantom{0}}  \end{array}  $
---	---

Observe, 1. If your divisor is 10, 100, 1000, &c. and your dividend an integer, cut off from the dividend as many decimal places as there are ciphers in the divisor, and you have the quot.

For 875 l. divided among 10 men, give 87.5 = 87 l. 10 s. to each.

	<i>£.</i>	<i>s.</i>	<i>d.</i>
10)875(87.5 = 87	10		
100)875(=8.75 = 8	15		
1000)875(.875 = 0	17	6	
10000)875(.0875 = 0	1	9	

† N

Observe,

## 146 INFINITE DECIMALS.

Observe, 2 If your divisor is 10, 100, 1000, &c. and your dividend has decimal places, augment the number of decimal places in the dividend by the number of ciphers in the divisor, and you have the quot. Thus, if 28 l. 15 s. = 28.75, are to be divided among 10 men, each man gets 2.875 = 2 l. 17 s. 6 d.

	£.	s.	d.	f.
10)28.75(2.875=2	17	6	0	
100)28.75(.2807=0	5	9	0	
1000)28.75(.02875=0	0	6	3	

## INFINITE DECIMALS.

**I**nfinite Decimals are such as repeat either the same figure, or the same circle of figures infinitely.

Hence they are distinguished into infinite repeating, and infinite circulating fractions.

Infinite repeating decimals arise from the first prime number 3, and its compounds, as denominators.

Infinite circulating decimals arise from the two prime numbers 7 and 11, and their composites.

I shall begin my Arithmetic of repeating decimals with one particular rule.

To continue any decimal of a pound of three places till it is limited, *i. e.* till it is determined whether it is finite or infinite : if the two figures farthest from the point are less than 24, multiply them by 4, the product (if less than 24) gives two places more in your decimal ; if the product is 24, or more, add 1 ; if 48, or more, add 2 ; if 72, or more, add 3 for the two following places. If the two figures farthest from the point exceed 25, 50, 75, multiply the excess by

# INFINITE DECIMALS. 147

4, as before ; continue this multiplication of the two last figures by 4, and adding as above, till you at last terminate either in 25, 50, or 75, and then your decimal is limited and finite ; or in a repeating 3, or a repeating 6, and then it is limited and infinite.

Directions for regulating infinite repeaters, are as follow.

1. They must not be limited under three decimal places.

2. Nor then, unless they have, at least, one repeating figure.

3. In Addition and Subtraction they must exceed the longest finite, by at least one place, and must all have the same number of places among themselves.

4. In Addition and Multiplication, for every 9 on the right hand, carry 1 to the next place ; and in Subtraction borrow 9 on the right hand.

## EXAMPLE OF ADDITION.

<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>f.</i>	<i>l.</i>
82	10	6	3=	82.528125
66	14	1	1=	66.7052083
72	15	4	2=	72.76875
98	10	7	3=	98.5322916
65	13	4	0=	65.6666666
79	18	8	0=	79.9333333
<hr/>				
£. 466	2	8	1=	466.1343750
<hr/>				

# 148 INFINITE DECIMALS.

## EXAMPLES of SUBTRACTION.

<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>f.</i>	<i>l.</i>
82	10	6	3	=82.528125
66	14	1	1	=66.7052083
<hr/>				
15	16	5	2	=15.8129166
<hr/>				

<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>f.</i>	<i>l.</i>
82	14	1	1	=82.7052083
25	10	7	3	=25.5322916
<hr/>				
57	3	5	2	=57.1729166
<hr/>				

## EXAMPLES of MULTIPLICATION.

27.8833
7.5
<hr/>
1394166
19518333
<hr/>
209.12500

79.666
500
<hr/>
39833.333

874.333
900
<hr/>
786900.000

84.777
800
<hr/>
67822.222

N. B.

N. B. If your multiplicand is infinite, and your multiplier has ciphers, (as in the three last examples), on the right hand, multiply the significant figures only; and instead of annexing the ciphers to the right hand of the product, repeat the right-hand figure of the product, as often as you have ciphers.

What is the price of 1000 stons of hay at 4 d. 1 f. per stone?

d. f.

$$\begin{array}{r}
 4 \quad 1 = .0177083 \quad .0177083 \\
 \qquad \qquad \qquad \qquad \qquad \qquad 1000 \\
 \hline
 \qquad \qquad \qquad \qquad \qquad \qquad \quad l. \quad s. \quad d. \\
 \qquad \qquad \qquad \qquad \qquad \qquad 17.7083333 = 17 \quad 14 \quad 2
 \end{array}$$

If the multiplier has a repeating infinite, set the repeating figures beyond the right-hand figure of the multiplicand, as we do with ciphers in the multiplication of integers; and if the repeating figure is 3, first take  $\frac{1}{3}$  of the multiplicand, and then multiply your other figures; if it is 6, take  $\frac{1}{3}$  of the multiplicand twice, and then multiply your other figures. but if it is any other than 3 or 6, first multiply that repeating figure into  $\frac{1}{3}$  of the multiplicand, and then multiply your other figures. In all these three cases allow no decimal place to the product for the repeating figure or figures in the multiplier.

EXAMPLES of each case.

$  \begin{array}{r}  3)728.25 \\  \underline{6.8333} \\  \frac{1}{3} = 24275 \\  582600 \\  436950 \\  \hline  496.375  \end{array}  $	Again,	$  \begin{array}{r}  3)796 \\  \underline{.2.6333} \\  \frac{1}{3} = 265333 \\  4776 \\  1592 \\  \hline  2096.1333  \end{array}  $
N 3		

# 150 INFINITE DECIMALS.

$$\begin{array}{r} 3)877.5 \\ \underline{0.333} \end{array}$$

$$\frac{1}{3}=292.5$$

$$\begin{array}{r} 3)368 \\ \underline{0.333} \end{array}$$

$$\frac{1}{3}=122.666$$

$$\begin{array}{r} 3)680.748 \\ \underline{3.21666} \end{array}$$

$$\frac{1}{3}=226.916$$

$$\frac{1}{3}=226.916$$

$$680.748$$

$$1361.496$$

$$2042.244$$

$$2189.73940$$

$$\begin{array}{r} 3)75.46 \\ \underline{2.4666} \end{array}$$

$$\frac{1}{3}=25.15333$$

$$\frac{1}{3}=25.15333$$

$$30184$$

$$15092$$

$$186.134666$$

$$\begin{array}{r} 9)7989 \\ \underline{47.555} \end{array}$$

$$\frac{1}{9}=887.666$$

$$5$$

$$4438333$$

$$55923$$

$$31956$$

$$379921.333$$

$$\begin{array}{r} 9)68.85 \\ \underline{0.111} \end{array}$$

$$\frac{1}{9}=7.65$$

What is the price of  $648\frac{1}{3}$  yards of cloth, at 13 s. 4 d. per yard?

$$\begin{array}{r} 3)648.5 \\ \underline{0.666} \end{array}$$

$$\frac{1}{3}=216.1666$$

$$\frac{1}{3}=216.1666$$

$$432.3333=132 \text{ l. } 6 \text{ s. } 8 \text{ d.}$$

# INFINITE DECIMALS. 151

In  $456\frac{1}{2}$  ducats, at 6 s. 2 d. each, how many pounds Sterling?

$$\begin{array}{r}
 3)456.5 \\
 \underline{.30833} \\
 \frac{1}{3}=152166 \\
 \quad 36520 \\
 \underline{136950} \qquad \text{l. s. d.} \\
 140.754166=140 \quad 15 \quad 1
 \end{array}$$

In  $10363\frac{1}{2}$  merks Scots, how many pounds Sterling?

N. B. 1 merk =  $1\frac{1}{4}$  l. Sterling = 055

$$\begin{array}{r}
 9)10363.5 \\
 \underline{.055} \\
 \frac{1}{9}=11515 \\
 \quad 5 \\
 \underline{\qquad} \quad \text{£. s.} \\
 575.75=575 \quad 15
 \end{array}$$

## DIVISION of INFINITE REPEATING DECIMALS.

If your divisor is finite, and your dividend infinite, in continuing your work, instead of annexing ciphers to the remainder, annex the repeating figure of the dividend.

E X.

# 152 INFINITE DECIMALS.

## EXAMPLES.

How many bolls of meal, at 5 l. 15 s. Scots each, will you buy for 561 l. 11 s. 8 d. Scots?

$$\begin{array}{r}
 \text{B.} \\
 5.75)561.583(97.666 \\
 \underline{5175} \\
 4408 \\
 \underline{4025} \\
 3833 \\
 \underline{3450} \\
 3833 \\
 \underline{3450} \\
 3833 \\
 \underline{3450} \\
 383
 \end{array}$$

If your divisor is infinite, in continuing your division, instead of annexing ciphers to the remainder, annex the right-hand figure to the remainder, as in the following

E X.

EXAMPLES.

Divide 684 l. 12 s. 8 d. among 5 men, so as 4 shall have equal shares, and the 5th only half a share.

$$4.5)684.633(152.1,407,4=152 \quad 2 \quad 1$$

$$\begin{array}{r}
 45 \\
 \hline
 234 \\
 225 \\
 \hline
 96 \\
 90 \\
 \hline
 63 \\
 45 \\
 \hline
 183 \\
 180 \\
 \hline
 333 \\
 315 \\
 \hline
 183 \\
 180 \\
 \hline
 (3)
 \end{array}$$

Divide

# 154 INFINITE DECIMALS.

Divide 486l. 16s. 8 d. among 4 men, so as three shall have equal shares, and the 4th only  $\frac{1}{4}$  of a share?

$$3.75)486.833(129.822=129 \text{ } \overset{\text{£.}}{16} \overset{\text{s.}}{5} \overset{\text{d.}}{5\frac{1}{2}}$$

375

1118

750

3683

3375

3083

3000

833

750

833

750

83

88 men divide 8469 l. 3 s. among them so as 87 of them shall have equal shares, and one of them only  $\frac{1}{3}$  share.

$$\begin{array}{r} \text{f.} \quad \text{s.} \quad \text{d.} \\ 87333 \overline{) 8469.150} (96.975 = 96 : 19 : 6 \\ 786000 \end{array}$$

$$\begin{array}{r} 609150 \\ 524000 \end{array}$$

$$\begin{array}{r} 851500 \\ 786000 \end{array}$$

$$\begin{array}{r} 655000 \\ 611333 \end{array}$$

$$\begin{array}{r} 43666 \\ 43666 \end{array}$$

# 156 INFINITE DECIMALS,

41 men in company gain 257 l. 13 s. 4 d. of which 39 are equal to have shares, and the other 2 are to have  $\frac{1}{2}$ , and the other  $\frac{1}{2}$  share :

		<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>f.</i>
	39.833)	257.666	(6.4686	=	6 : 9 : 4 : 2
$\frac{1}{2} = .5$		239000			
$\frac{1}{3} = .333$			186666		
			159333		
			273333		
			239000		
			343333		
			318666		
			246606		
			239000		
			.7666		

# INFINITE DECIMALS. 157

A man left his estate, amounting to 4765 l. 18 s. 8 d. among 12 relations, to be divided in the following manner; viz. 6 of them were to have equal shares, 4 of them  $\frac{1}{3}$  share each, and two of them  $\frac{1}{4}$  share each: *Quer.* how much each person received thereof?

$$\begin{array}{r}
 \frac{1}{4} = .25 \text{ and } \frac{1}{3} = .333 \\
 \underline{2} \qquad \qquad \qquad \underline{4} \\
 .50 \qquad \qquad \qquad 1.333 \\
 \qquad \qquad \qquad \underline{.50} \\
 \qquad \qquad \qquad 1.833 \\
 \qquad \qquad \qquad \underline{6} \\
 \qquad \qquad \qquad 7.833
 \end{array}
 \qquad
 \begin{array}{r}
 7.833 \overline{) 4765.933(608.417} \\
 \underline{47000} \\
 65933 \\
 \underline{62666} \\
 32666 \\
 \underline{31333} \\
 13333 \\
 \underline{7833} \\
 55000 \\
 \underline{54833} \\
 166 = 2 f.
 \end{array}$$

$$\begin{array}{r}
 6 \text{ get } 3650 \text{ } 10 \text{ } 0 \text{ } 2 \\
 4 \qquad \quad 811 \text{ } 4 \text{ } 5 \text{ } 0 \\
 2 \qquad \quad 304 \text{ } 4 \text{ } 2 \text{ } 2 \\
 \hline
 \end{array}$$

|Proof £. 4765:18:8 0

In case your divisor is an infinite, repeating the same simple figures, as 111, .333, .555; take the following notandum

† O

N. B.

# 158 INFINITE DECIMALS.

N. B. 1. If your divisor is .333, multiply the dividend by 3, and you have the quot.

Thus,  $468.433 \div .333 = 1405.3$

$$\begin{array}{r} .333 \overline{) 468.433} \\ \underline{\phantom{.}3} \\ 1405.300 \end{array}$$

N. B. 2. If your divisor is .666, multiply the dividend by 3, and divide that product by 2, and you have the quot.

Divide 432.333 by .666

$$\begin{array}{r} .666 \overline{) 432.333} \\ \underline{\phantom{.}3} \\ 2 \overline{) 1297.000} \\ \underline{\phantom{.}6} 48.5 = 648 \text{ l. } 10 \text{ s.} \end{array}$$

Or multiply by 1.5, which is the same thing as  $\frac{3}{2}$ ?

$$\begin{array}{r} 432.333 \\ \underline{1.5} \\ 2161.666 \\ \underline{4323.333} \\ 648.5000 \end{array}$$

N. B.

## INFINITE DECIMALS. 159

N. B. 3. If the divisor is .111, multiply the dividend by 9, and you have the quot.

Divide 76.074074 by .111

.111) 76.074074  
          9              *l. s. d.*  
684.666666 = 684 13 4

If your divisor is any other repeating figure (than .333, 666, .111), as .555, multiply the dividend by 9, and divide the product by the repeating figure, and you have the quot.

Thus, divide 836.333 by .555.

$$\begin{array}{r} 836.333 \\ 9 \\ \hline 5) 7527.000 \quad l. s. \\ \hline 1505.4 = 1505 \quad 8 \end{array}$$

Or multiply the dividend by 1.8, which is the same with  $\frac{9}{5}$ ?

$$\begin{array}{r} 836.333 \\ 1.8 \\ \hline 6690666 \\ 8363333 \\ \hline \end{array}$$

*l. s.*  
 $1505.4000 = 1505 \text{ 8, and so of all the rest.}$

**O 2**

We

## 160. INFINITE DECIMALS.

We should now treat of circulating decimals; but their process being extremely tedious, they would swell this treatise beyond what was originally intended, and besides we reckon enough has been advanced to illustrate decimals in general. However, such as incline to study them, will find them fully treated of either by Mr Cunn or Mr Wright in their respective treatises upon fractions; therefore we shall conclude the doctrine of decimals, by a few examples, which will serve to illustrate the rule of Proportion of decimals. And herein it will suffice to observe, that all the fractional parts must be brought into decimals, and then proceed with the operation according to the rules already laid down for Proportion.

### E X A M P L E S.

If I pay 3 l. 16s. for  $18\frac{1}{2}$  yards of cloth, what will  $32\frac{3}{4}$  yards of that cloth amount to?

$$\begin{array}{r}
 y. \quad \quad \quad y. \\
 18.5 : 3.8 :: 32.75 \\
 \quad \quad \quad 3.8 \\
 \hline
 \quad \quad \quad 26200 \\
 \quad \quad \quad 9825 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 l. \quad s. \quad d. \\
 18.5 \quad 124450 \quad (6.7271. = 6 \quad 14 \quad 6\frac{1}{2}
 \end{array}$$

P R O.

# PROPORTION of DECIMALS. 165

## PROPORTION of DECIMALS.

**W**HAT must the penny-loaf weigh, when wheat is at 10 s. per boll, if the said loaf weighs  $5\frac{1}{2}$  ounces, when wheat is at 6 s. 8 d. per boll?

$$\begin{array}{r}
 \text{1. oz. 1.} \\
 .333 : 5.5 :: .5 \\
 \underline{5.5} \\
 1666 \\
 16666 \\
 \hline
 \text{oz. pwt. gr.} \\
 .5) 1.8333 \quad (3.666 = 3 \text{ } 13 \text{ } 7 \\
 \underline{15} \\
 33 \\
 \underline{30} \\
 33 \\
 \underline{30} \\
 33 \\
 \underline{30} \\
 (3)
 \end{array}$$

# 162 PROPORTION of DECIMALS.

If the carriage of  $\frac{1}{2}$  Cwt. for 40 miles cost 6 d.  
what will the carriage of  $16\frac{1}{4}$  Cwt. for 100 miles amount to ?

Cwt.      l. Cwt.

$$.5 : .025 :: .16.25$$

.025

8125

3250

$$.5) .40625 (.8125$$

Then      m.      l.      m.

$$40 : .8125 :: 100$$

100

l. s. d.

$$40) 81.2500 (2.03125 = 2 \text{ } 0 \text{ } 7\frac{1}{2}$$

40

125

120

50

40

100

80

200

200

If

# PROPORTION of DECIMALS. 163

If the beam of a balance be 81 inches long, and cheats at the rate of 7.5 l. per cent. how much is the one end shorter than the other?

$$\begin{array}{r} 107.5 \\ 100 \\ \hline \end{array}$$

$$207.5 : 81 :: 100$$

$$\begin{array}{r} 81 \\ \hline \end{array}$$

$$207.5) 8100.00 \text{ (39.0361445 inches for the (shortest end.)}$$

$$\begin{array}{r} 1625 \\ \hline \end{array}$$

$$207.5 : 81 :: 107.5$$

$$\begin{array}{r} 81 \\ \hline 1075 \\ 8600 \\ \hline \end{array}$$

$$207.5) 87075 \text{ (41.9638554 longest end}$$

$$\begin{array}{r} 39.0361445 \text{ shortest} \\ \hline 450 \end{array}$$

In. 2.9277109 Ans

## DECIMAL TABLES.

Coin.		Avoirdupois.
1 l. Sterling the integer.	112 lb. the integer.	
.05=1 s.	.25=1 qr.	
.004166=1 d.	.008928=1 lb.	
.0010416=1 f.	.000558=1 oz.	
	.000034=1 dram	
		Troy

*Troy Weight.*

1 lb. the integer.

 $.08333=1$  oz. $.004166=1$  pwt. $.000173=1$  gr.*Time.*

1 year the integer.

 $.002739=1$  day $.000114=1$  hour $.0000019=1$  minute*Time.*

1 day the integer.

 $.041666=1$  hour $.000694=1$  minute $.00001157=1$  second*Cloth measure.*

1 yard the integer.

 $.25=1$  qr. $.0625=1$  nail*Liquid measure.*

1 gallon the integer.

 $.125=1$  pint $.0625=1$  chop. $.0078125=1$  gill*Corn measure.*

1 boll the integer.

 $.25=1$  firlo $.0625=1$  peck $.015625=1$  lippie*Land measure.*

1 acre the integer.

 $.25=1$  rood $.00625=1$  fall $.0001736=1$  ell

The use of these tables will be very evident by the following

## E X A M P L E S.

It is required to find the decimal parts equivalent to 17 s. 9 d. 2 farthings.

$.05=1$  s. therefore  $17 \times .05=.85=17$  s.

$.004166=1$  d. therefore  $.004166 \times 9=.037494=9$  d.

$.0010416=1$  f. therefore  $.0010416 \times 2=.002083=2$  f.

Consequently their sum is  $.889577=17$  s. 9 d. 2 f.

What

What is the sum of  $\frac{1}{2}$  of  $\frac{1}{4}$  of  $\frac{1}{2}$  of a shilling, when added to  $\frac{1}{4}$  of a guinea? Note, a guinea the integer.

$\frac{1}{2}$  of  $\frac{1}{4}$  of  $\frac{1}{2} = \frac{1}{8}$ , which reduced to a decimal, is .025, and .025 of a shilling divided by 21, quotes .00119 of a guinea, and  $\frac{1}{4}$  of a guinea is .875.

$$\begin{array}{r} .875 \\ .00119 \\ \hline \end{array} \quad \begin{array}{l} s. \quad d. \quad f. \\ \text{sum } .87619 = 17 \quad 4 \quad 1 \end{array}$$

What is the sum of  $\frac{1}{4}$  of 2 l. and  $\frac{1}{2}$  of 30 shillings?

$\frac{1}{4}$  of 2 l. = .75, and  $\frac{1}{2}$  of 30 s. = 6

$$\begin{array}{r} .75 \\ 40 \\ \hline 30.00 \end{array}$$

$$\begin{array}{r} .6 \\ 30 \\ \hline 18.0 \\ 30 \\ \hline \end{array}$$

sum 48 shillings.

What is the sum of  $\frac{1}{4}$  of a crown, and  $\frac{1}{2}$  of a pound Sterling?

$\frac{1}{4}$  of a crown = .875, which divided by 4 quotes .21875 of a l.

and  $\frac{1}{2}$  of a pound = .8

$$\begin{array}{r} .21875 \\ \hline \end{array} \quad \begin{array}{l} f. \quad s. \quad d. \\ 1.01875 = 1 \quad 0 \quad 4\frac{1}{2} \end{array}$$

What

What is the difference betwixt  $\frac{3}{4}$  of a guinea, and  $\frac{3}{4}$  of a crown?

$$\begin{array}{r} \frac{3}{4} = .75 \\ 5 \\ \hline 3.75 \end{array}$$

$$\begin{array}{r} \text{and } \frac{1}{4} = .25 \\ 21 \\ \hline 25 \end{array}$$

$$\begin{array}{r} 50 \\ \hline 5.25 \\ 3.75 \\ \hline \end{array}$$

$$1.50 = 1 \text{ s. } 6 \text{ d.}$$

What is the difference betwixt  $\frac{1}{4}$  of a pint and  $\frac{1}{4}$  of a gill?

$$\frac{1}{4} \text{ of a gill} = .75, \text{ and } \frac{1}{4} \text{ of a pint} = .375$$

16 gills in a pint.

$$\begin{array}{r} 6.000 \\ 75 \\ \hline \end{array}$$

$$75$$

Difference 5.25 gills.

What is the difference betwixt  $\frac{7}{8}$  of 2 ells and  $\frac{1}{8}$  of a fall running measure?

$$\begin{array}{r} \frac{7}{8} = .875 \\ 2 \\ \hline \end{array}$$

$$\begin{array}{r} 1.750 \\ 1.2 \\ \hline \end{array}$$

$$.55 = 2.2 \text{ qrs}$$

$$\begin{array}{r} \text{and } \frac{1}{8} = .2\frac{1}{2} \\ 6 \\ \hline \end{array}$$

$$1.2$$

EXTRACTION of the SQUARE  
R o o t.

**T**O extract the Square Root, is to find the side of a square figure; but numerically speaking, it is to find out such a number, which multiplied into itself, will produce the number given.

Thus the square root of 64 is 8, and 8 times 8 is 64.

**RULE** for extracting the square root, point under the figure on the right hand, and then pass over one figure, and thus point on till you have pointed off all your figures, remembering always to pass one; then find the root of the figure or figures of the first point towards the left hand, and set it down by way of a quotient; and it being multiplied into itself, set it down under the first point, and subtract them; then to the remainder you will subjoin the two figures of the next period; next double your quotient, and set it down by way of a new divisor, and then ask how oft you will get this new divisor out of your last period and remainder, setting down, both in your quotient and divisor, the number of times you can get this new divisor; then multiply your divisor by the last found figure in the quot, setting down the product under your period, and then subtract, and thus proceed unto the end.

# 168 EXTRACTION of the

## A TABLE of ROOTS or POWERS.

Roots or 1st powers.	1	2	3	4	5	6	7	8	9
Squares or 2d powers.	1	4	9	16	25	36	49	64	81
Cubes or 3d powers.	1	8	27	64	125	216	343	512	729
Biquadrates or 4th powers.	1	16	81	256	625	1296	2401	4096	6561

N. B. Numbers whose roots are to be extracted are twofold.

1. Squares, whose root is exactly found out without any remainder.

2. Surds, whose root cannot be found out without some remainder.

# S Q U A R E R O O T. 169

## E X A M P L E S.

Extract the square root of 121.

$$\begin{array}{r} 121 \overline{)11} \\ 1 \\ \hline 21 \overline{)21} \\ 21 \\ \hline \end{array}$$

I demand the square root of 7225.

$$\begin{array}{r} 7225 \overline{)85} \\ 64 \\ \hline 165 \overline{)825} \\ 825 \\ \hline \end{array}$$

What is the square root of 15625?

$$\begin{array}{r} 15625 \overline{)125} \\ 1 \\ \hline 22 \overline{)56} \\ 44 \\ \hline 245 \overline{)125} \\ 1225 \\ \hline \end{array}$$

What is the square root of 50384985156?  
Ans. 224466.

† P

What

## 170 EXTRACTION of the

What is the square root of 4712.81261? *Ans.* 68.649.

What is the square root of 3.1721812? *Ans.* 1.78106.

What is the square root of .0007612816? *Ans.* .02759.

If it is required to extract the square root of a vulgar fraction;

**RULE.** First reduce the fraction to its lowest terms, and then extract the square root of the numerator for a new numerator, and the square root of the denominator for a new denominator; but if the fraction be a surd, reduce it to a decimal, and then extract the square root of it; but be sure that the decimal consist of an even number of places, as two, four, six, &c.

## EXAMPLES.

What is the square root of  $\frac{3044}{819}$ ? *Ans.*  $\frac{2}{3}$ .

For the greatest common measure of  $\frac{3044}{819}$  is 761, therefore  $\frac{761}{761})\frac{3044}{819}=\frac{4}{9}$ , the square root of which is  $\frac{2}{3}$ .

What is the square root of  $\frac{3456}{3136}$ ? *Ans.*  $\frac{4}{7}$ .

What is the square root of  $\frac{7050}{9118}$ ? *Ans.*  $\frac{7}{8}$ .

## SURDS.

What is the square root of  $\frac{3168}{819}$ ? *Ans.* .71528.  
+remainder.

If a mixt number is given, to extract the root thereof,

**RULE.** Reduce the fractional part of the mixt number to its lowest terms, and then reduce the mixt number to an improper fraction, and extract

tract the roots of the numerator and denominator; but if it be a surd, reduce the fractional part to a decimal, and annex it to the whole number, and then extract the root.

What is the square root of  $37\frac{3}{4}$ ? *Ans.*  $6\frac{1}{2}$ .

What is the square root of  $76\frac{1}{4}$  a surd? *Ans.*  $8.7649\frac{1}{2}$  remainder.

## EXTRACTION of the CUBE ROOT.

**A** Cube number is that which is contained under three equal numbers, or which is equally equal.

So 8 is a cube number contained under 3 equal numbers, to wit, 2, 2 and 2, for 2 times 2 is 4, and 2 times 4 is 8; and the cube number 27 is contained under 3, 3 and 3, for 3 times 3 is 9, and 3 times 9 is 27; and so of the rest, as in the table, page 168. where we have inserted the cubes with their genitive equal numbers as far as the nine digits. And when it is required to extract the cube root of any given number, we have nothing to do, but to find that equal number of which it is composed; so if the root of 64 was required, it would be found to be 4, as in the table; here 4 is the root, or first power, and 4 times 4 is 16, the 2d power, and 4 times 16 is 64, the 3d power or cube.

Of cube numbers to be extracted there are three sorts.

1st, Single.

2dly, Compound.

3dly, Irrational.

Single are all such cubes as are composed or made up of any of the 9 digits, of which sort are those in the foregoing table above referred to.

Compound are such cubes as are composed of

## 172 E X T R A C T I O N of the

more figures than one, as 1000, whose root is 10; 1331, whose root is 11; 1728, whose root is 12, &c.

Irrational are all such cubes, whose root cannot be discovered by art exactly, neither in whole numbers nor fractions, but something will still remain; there being no proportion yet found betwixt an irrational or surd number, and its root; such numbers are 5, 7, 36, 160, 1526, &c.

The extraction of the cube root participates something of division, yet much more difficult.

The root of any single cube number is found by inspection, as may be seen in the foregoing table.

But if it be a compound cube number, it must be prepared by pointing thus; make a point above or under the place of units, and then omitting two figures, point every third figure, and as many points as your numbers contain, so many figures will your root consist of.

**R U L E 1.** Find the root of your period toward the left hand, which root place by way of a quotient.

2. Place the cube of the root found as above, below your first period, which cube must be subtracted from said period, and then bring down your second period, and annex it to the remainder, termed by some the resolvend or dividend.

3. Divide said resolvend by just 300 times the square of the number in the quot; and then ask how oft this divisor is contained in your resolvend, and place the number of times in your quot; then multiply said divisor by the last figure in your quot, and place this product below said resolvend, with a line betwixt them.

4. Square the last found figure in your quot, and multiply it by the other figure or figures therein,

therein, the product of which, when multiplied by 30, you'll place below your last line.

5. Then place the cube of your last-found figure in the quot, below these two lines; which three being added, subtract their sum from the resolvend; and thus proceed till all your periods are taken down. Note, that these three lines are called by some subtrahends, and by others subducends.

## E X A M P L E S.

Extract the cube root of 46656.

$$\begin{array}{r}
 46656(36 \\
 \underline{27} \\
 2700)19656 \text{ resolvend or dividend.} \\
 \begin{array}{r}
 16200 \\
 3240 \\
 216
 \end{array}
 \left. \vphantom{\begin{array}{r} 16200 \\ 3240 \\ 216 \end{array}} \right\} \text{ subtrahend or subducend.} \\
 \hline
 19656 \\
 \hline
 (0)
 \end{array}$$

# 174 EXTRACTION of the

What is the cube root of 673373097125?

$$\begin{array}{r}
 \begin{array}{r}
 \overset{\cdot}{6}\overset{\cdot}{7}\overset{\cdot}{3}\overset{\cdot}{3}\overset{\cdot}{7}\overset{\cdot}{3}\overset{\cdot}{0}\overset{\cdot}{9}\overset{\cdot}{7}\overset{\cdot}{1}\overset{\cdot}{2}\overset{\cdot}{5} \\
 \underline{512} \\
 19200) 161373 \quad \text{1st Dividend.} \\
 \underline{134400} \\
 11760 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Subducends.} \\
 343 \\
 \hline
 \text{Sum } 146503 \\
 2270700) 14870097 \quad \text{2d Dividend.} \\
 \underline{13624200} \\
 93960 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Subducends.} \\
 216 \\
 \hline
 \text{Sum } 13718376 \\
 230212800) 1151721125 \quad \text{3d Dividend.} \\
 \underline{1151064000} \\
 657000 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Subducends.} \\
 125 \\
 \hline
 1151721125 \\
 \hline
 (0)
 \end{array}
 \end{array}$$

What

What is the cube root of 444194.947 (76.3

$$\begin{array}{r}
 .343 \\
 \hline
 14700) 101194 \\
 \hline
 88200 \\
 7560 \\
 216 \\
 \hline
 95976 \\
 \hline
 1732800) 5218.947 \\
 \hline
 5198400 \\
 20520 \\
 27 \\
 \hline
 5218947 \\
 \hline
 (0)
 \end{array}$$

N. B. If any thing remain after having extracted the cube root of any number, you may add 3, 6, or 9 ciphers, and proceed as before : and if you are to extract the cube root of a decimal, the foregoing rules must be practised, just as in any whole number.

### EXAMPLE.

What is the cube root of .000141600?

$$\begin{array}{r}
 .000141600(.052 \text{ root required.} \\
 \hline
 \text{Rem. 992}
 \end{array}$$

If a vulgar fraction or mixt number be commensurable to its root, prepare it the same way as prescribed in the square root, and then extract its cube root.

## E X A M P L E S.

What is the cube root of  $\frac{252}{1188}$ ? *Ans.*  $\frac{2}{3}$ .

What is the cube root of  $\frac{1944}{3888}$ ? *Ans.*  $\frac{3}{4}$ .

But if it is a surd, it must be reduced to a decimal, thus .763 is the cube root of  $\frac{4}{9}$ .

THE C U B E AND S Q U A R E  
R O O T S R E D U C E D T O P R A C -  
T I C E.

**T**Here is an army consisting of a certain number of men who are placed in rank and file, *i. e.* in the form of a square, each side having 472 men; I demand how many men the whole square contains?

$$\begin{array}{r}
 472 \\
 472 \\
 \hline
 944 \\
 3304 \\
 1888 \\
 \hline
 222784
 \end{array}$$

The

The floor of a certain great room is made exactly square, each side of which contains 75 feet; I demand how many square feet are therein ?

$$\begin{array}{r}
 75 \\
 75 \\
 \hline
 375 \\
 525 \\
 \hline
 5625
 \end{array}$$

Suppose 12544 foldiers are to be placed rank and file in the form of an exact square, I demand how many will be in the front, and how many in depth ?

$$\begin{array}{r}
 \dot{1} \dot{2} \dot{5} \dot{4} \dot{4} (112 \\
 \underline{1} \\
 21 \overline{)25} \\
 \underline{21} \\
 222 \overline{(444} \\
 \underline{444}
 \end{array}$$

A certain square pavement contains 197136 square stones, all of them of the same size; I demand

mand how many are contained in one of its sides?

$$\begin{array}{r}
 197136(444 \\
 16 \\
 \hline
 84)371 \\
 336 \\
 \hline
 884)3536 \\
 3536 \\
 \hline
 \end{array}$$

The wall of a town being 17 feet high, which is surrounded with a mote of 20 feet in breadth, I demand the length of a ladder that will reach from the outside of the mote to the top of the wall?

$$\begin{array}{r}
 17 \quad 20 \\
 17 \quad 20 \\
 \hline
 289 + 400 \\
 289 + 289 \\
 \hline
 689
 \end{array}
 \qquad
 \begin{array}{r}
 689)26.248 \\
 4 \\
 \hline
 46)289 \\
 276 \\
 \hline
 522)1300 \\
 1044 \\
 \hline
 5244)25600 \\
 20976 \\
 \hline
 52488)462400 \\
 419904 \\
 \hline
 42496
 \end{array}$$

Suppose

# S Q U A R E   R O O T S.   179

Suppose a cellar to be dug that shall be 12 feet every way, in length, breadth, and depth; how many solid feet of earth must be taken out to complete the same? *Ans.* 1728 feet.

If a cubical piece of timber be 41 inches long, 41 inches broad, and 41 inches thick; how many cubical inches does it contain? *Ans.* 68921 inches.

Suppose a stone of a cubical form to contain 474552 inches, what is the superficial content of one of its sides?

$$\begin{array}{r}
 47 \cdot 1552 (78 \times 78 = 6084 \text{ inch. } \textit{Ans.} \\
 \underline{343} \\
 14700 \overline{)131552} \\
 \underline{117600} \\
 13440 \\
 \underline{512} \\
 131552
 \end{array}$$

If a bullet of brass of 8 inches diameter weigh 72 lb. what will a bullet of brass weigh whose diameter is 4 inches?

$$\begin{array}{r}
 512 : 72 :: 64 \\
 \underline{64} \\
 288 \\
 \underline{432} \\
 512 \overline{)4608} (9 \text{ lb. } \textit{Ans.} \\
 \underline{4608}
 \end{array}$$

## 180 COMPOUND INTEREST.

If a ship of 100 tun be 44 feet long at the keel, of what length will the keel of a ship be of 220 tuns?

The cube of 44=85184  
 $100 : 85184 :: 220$

$$\begin{array}{r} 220 \\ \hline 1703680 \\ 170368 \\ \hline \end{array}$$

1,00)187404,80(57.225 feet *Ans.*  
 125

$$\begin{array}{r} 7500)62404 \\ \hline 52500 \\ 7350 \\ 343 \\ \hline 60193 \\ \hline 2211800 \end{array}$$

## COMPOUND INTEREST.

**W**hen the amount of any sum at compound interest is required; **RULE**, find the amount of 1 l. for any number of years at any rate *per cent.* by which multiplying the sum proposed, the product is the answer.

Thus, interest being at 5 *per cent.*  
 years.

1  $100 : 105 :: 1 : 1.05$

2  $100 : 105 :: 1.05 : 1.1025$

3  $100 : 105 :: 1.1025 : 1.157625$

4  $100 : 105 :: 1.157625 : 1.21550625$

The

# COMPOUND INTEREST. 181

The letters made use of in compound interest are,

A, the amount,

P, the principal,

T, the time,

R, the amount of 1 l. for 1 year, at any given rate, which is found as above.

A TABLE of the AMOUNTS of 1 l. for 1 year.

Rates per cent.	Amounts of 1 l.	Rates per cent.	Amounts of 1 l.	Rates per cent.	Amounts of 1 l.
3	1.03	5½	1.055	8	1.08
3½	1.035	6	1.06	8½	1.085
4	1.04	6½	1.065	9	1.09
4½	1.045	7	1.07	9½	1.095
5	1.05	7½	1.075	10	1.1

+ 2

181,

# 182 COMPOUND INTEREST.

1st, When P, T, and Rate are given, to find A.

RULE.  $p \times r = a$ .

## EXAMPLES.

What will 225 l. amount to in 3 years time, at 5 per cent. per annum? *Ans.*  $1.05 \times 1.05 \times 1.05 = 1.157625$ , then  $1.157625 \times 225 = 260 \text{ l. } 9 \text{ s. } 3\frac{3}{4} \text{ d.}$

What will 200 l. amount to in 4 years, at 5 per cent. per annum? *Ans.* 243 l. 2.25 s.

What will 450 l. amount to in 5 years, at 4 per cent. per annum? *Ans.* 547 l. 9 s. 10 d. 2.0538368 f.

What will 500 l. amount to in 4 years, at  $5\frac{1}{2}$  per cent. per annum? *Ans.* 619 l. 8 s. 2 d. 3.8323 f.

2dly, When A, R, and T are given, to find P.

RULE.  $\frac{a}{r} = P$ .

## EXAMPLES.

What principal being put out to interest will amount to 260 l. 9 s. 3 d. 3 f. at 5 per cent. per annum, for 3 years? *Ans.*  $1.05 \times 1.05 \times 1.05 = 1.157625$ ; then,

$$\frac{260.465625}{1.157625} = 225 \text{ l.}$$

What principal being put out to interest will amount to 243 l. 2.025 s. in 4 years, at 5 per cent. per annum? *Ans.* 200 l.

What principal will amount to 547 l. 9 s. 10 d. 2.0538368 f. in 5 years, at 4 per cent. per annum? *Ans.* 450 l.

What principal will amount to 619 l. 8 s. 2 d. 38.323 f. in 4 years, at  $5\frac{1}{2}$  per cent. per annum? *Ans.* 500 l.

3dly, When P, R, and T are given, to find R.

RULE.

# COMPOUND INTEREST. 183

**RULE.**  $\frac{a}{P} = r$  which being extracted by the rules of extraction, (the time given in the quotient shewing the power), will give R.

## EXAMPLES.

At what rate *per cent.* will 225 l. amount to 260 l. 9 s. 3 d. 3 f. in 3 years?

$$\text{Ans. } \frac{260.565625}{225} = 1.157625$$

the cube root of which (it being the third power)  $= 1.05 = 5 \text{ per cent.}$

At what rate *per cent.* will 200 l. amount to 243 l. 2.025 s. in 4 years? *Ans.* 5 *per cent.* (it being the fourth power).

At what rate *per cent.* will 450 l. amount to 547 l. 9 s. 10 d. 2.0538368 f. in 5 years? *Ans.* at 4 *per cent.*

At what rate *per cent.* will 500 l. amount to 619 l. 8 s. 2 d. 3.8323 f. in 5 years? *Ans.* at 5  $\frac{1}{2}$  *per cent.*

4thly, When P, A, R are given, to find T.

**RULE.**  $\frac{a}{P} = r$  which being continually divided by  $r$ , till nothing remains, the number of those divisions will be equal to T.

## EXAMPLES.

In what time will 225 l. amount to 260 l. 9 s. 3 d. 3 f. at 5 *per cent.*? *Ans.*  $\frac{260.465625}{225} = 1.157625$ , then

$$\frac{1.157625}{1.05} = 1.1025, \text{ and } \frac{1.1025}{1.05} = 1.05 \quad \frac{1.05}{1.05} = 1 \text{ the}$$

Q 2

number

## 84 COMPOUND INTEREST.

number of divisions being 3 = time sought.

In what time will 200 l. amount to 243 l. 2.025 s. at 5 *per cent. per annum*? *Ans.* 4 years.

In what time will 450 l. amount to 547 l. 9 s. 10 d. 2.0538368 f. at 4 *per cent.*? *Ans.* 5 years.

In what time will 500 l. amount to 619 l. 8 s. 2 d. 3.8323 f. at  $5\frac{1}{2}$  *per cent.*? *Ans.* 4 years.

## ANNUITIES or PENSIONS in ARREARS.

Note, U represents the annuity, pension, or yearly rent; A, R, T, as before.

1st, When U, T, R are given, to find A.

$$\text{RULE. } \frac{ur - u}{r - 1} = A$$

Multiply the amount of 1 l. for the number of years, and at the rate *per cent.* given in the question, by the annuity, pension, &c. and it will give the answer.

## EXAMPLES.

What will an annuity of 50 l. *per annum* payable yearly, amount to in 4 years, at 5 *per cent.*?

*Ans.*  $1.05 \times 1.05 \times 1.05 \times 1.05 \times 50 = 60.77531250$

then  $\frac{60.77531250 - 50}{1.05 - 1} = 215 \text{ l. } 10 \text{ id. } 3 \text{ far.}$

What will a pension of 45 l. *per annum* payable yearly, amount to in 5 years, at 5 *per cent.*?

*Ans.* 248 l. 13 s. 0 d. 3.27 far.

If a salary of 40 l. *per annum* to be paid yearly,

bc

# COMPOUND INTEREST. 185

be forborn 6 years, at 6 *per cent.* what is the amount? *Ans.* 279 l. 0 s. 3.072 d.

If an annuity of 75 l. *per annum* payable yearly, be omitted to be paid for 10 years, at 6 *per cent.* what is the amount? *Ans.* 988 l. 11 s. 2.22 d.

2dly, When U, T, R are given, to find U.

RULE.  $\frac{ar-a}{r-1} = U$

## EXAMPLES.

What annuity being forborn 4 years will amount to 215 l. 10 s. 1 d. 2 far. at 5 *per cent.*?

*Ans.*  $\frac{215.50525 \times 1.05 - 215.50625}{1.05 \times 1.05 \times 1.05 \times 1.05 - 1} = 50 \text{ l.}$

What pension being forborn 5 years will amount to 248 l. 13 s. 0 d. 3.27 far. at 5 *per cent.*?

*Ans.* 45 l.

What salary being omitted to be paid 6 years, will amount to 279 l. 0 s. 3.072 d. at 6 *per cent.*?

*Ans.* 40 l.

If the payment of an annuity being forborn 10 years, amount to 988 l. 11 s. 2.22 d. at 6 *per cent.* what is the annuity? *Ans.* 75 l.

3dly, When U, A, R are given, to find T.

RULE  $\frac{ar+u-a}{u} = r$  which being continually divided by R, till nothing remains, the number of those divisions will be equal to T.

# 186 COMPOUND INTEREST.

## EXAMPLES.

In what time will 50 l. *per annum* amount to 215 l. 10 s. 1 d. 2 far. at 5 *per cent*, for non-payment?

*Ans.*  $\frac{215.50625 \times 1.05 + 50 - 215.50625}{50} = 1.21550625$ ; which being continually divided by R, the number of those divisions will be equal to 4 years.

In what time will 45 l. *per annum* amount to 248 l. 13 s. 0 d. 3.27 far. allowing 5 *per cent*. for forbearance of payment? *Ans.* 5 years.

In what time will 40 l. *per annum* amount to 279 l. 0 s. 3.072 d. at 6 *per cent*.? *Ans.* 6 years.

In what time will 75 l. *per annum* amount to 988 l. 11 s. 2.22 d. allowing 6 *per cent*. for forbearance of payment? *Ans.* 10 years.

## PRESENT WORTH OF ANNUITIES, PENSIONS, &c.

1. **W**hen U, T, R are given, to find P.

RULE.  $\frac{U - R}{R - 1} = P.$

E X.

# COMPOUND INTEREST. T. 187

## EXAMPLES.

What is the present worth of an annuity of 30 l. *per annum*, to continue 7 years, at 6 *per cent.*?

$$\text{Ans. } \frac{30}{1.50363} = 19.9517 \quad 30 - 19.9517 = 10.4083$$

then  $\frac{10.4083}{1.06-1} = 1467 \text{ l. } 9 \text{ s. } 5.184 \text{ d.}$

What is the present worth of a pension of 40 l. *per annum*, to continue 8 years, at 5 *per cent.*?

*Ans.* 258 l. 10 s. 6 d. 3.264 f.

What is the present worth of a salary of 35 l. to continue 7 years, at 6 *per cent.*?

*Ans.* 195 l. 7 s. 7 d. 3.968 f.

What is the yearly rent of 20 l. to continue 6 years, worth in ready money, at 5 *per cent.*?

*Ans.* 101 l. 10 s. 3 d. 1.248 f.

2dly, When P, T, R are given, to find U.

$$\text{RULE, } \frac{pr \times r - pr}{r - 1} = U.$$

## EXAMPLES.

If an annuity be purchased for 167 l. 9 s. 5.184 d. to be continued 7 years, at 6 *per cent.*, what is the annuity?

$$\text{Ans. } \frac{167.4716 \times 1.50363 \times .06 - 167.4716 \times 1.50363}{1.50363 - 1} = 30 \text{ l.}$$

If

# 188 COMPOUND INTEREST.

If the present payment of 258 l. 10. 6 d. 3.264 f. be made for a salary 8 years to come, at 5 *per cent.* what was the salary? *Ans.* 40 l.

If the present payment of 195 l. 7 s. 7 d. 3.968 f. were required for a pension for 7 years to come, at 5 *per cent.* what is the pension? *Ans.* 35 l.

If the present worth of an annuity 6 years to come, be 101 l. 10 s. 3 d. 1.248 f. at 5 *per cent.* what is the annuity? *Ans.* 20 l.

3dly, When U, P, R are given, to find T.

**RULE.**  $\frac{U}{P \times U - PR} = r$  which being continually divided by R, till nothing remains, the number of those divisions will be equal to T.

## EXAMPLES.

How long may a lease of 30 l. of yearly rent be had for 167 l. 9 s. 5.184 d. allowing 6 *per cent.* to the purchaser? *Ans.*  $\frac{30}{167.4716 + 30 - 177.5198} = 1.05363$

which being continually divided, the number of those divisions will be 7 equal to T.

If 258 l. 10 s. 6 d. 3.264 f. be paid down for a lease of 40 l. *per annum* at 5 *per cent.* how long is the lease purchased for? *Ans.* 8 years.

If a house is let upon lease for 35 l. *per annum*, and the lessee makes present payment of 195 l. 7 s. 8 d. he being allowed 6 *per cent.* I demand how long the lease is purchased for? *Ans.* 7 years.

For what time may a lease of 26 l. be purchased, when present payment is made of 10 l. 10. s. 3 d. 2 f. at 5 *per cent.*? *Ans.* 6 years.

R E.

# REBATE or DISCOUNT. 189

## REBATE or DISCOUNT.

1st, **W**HEN S, T, R are given, to find P.

RULE.  $\frac{S}{r} = P$

### EXAMPLES.

What is the present worth of 315 l. 12 s. 4 d. payable 4 years hence, at 6 per cent.? *Ans.*  $1.06 \times 1.06 \times 1.06 \times 1.06 = 1.26247$  then  $\frac{315.6175}{1.26247} = 250$  l.

If 344 l. 14 s. 9 d. 1.92 far. be payable in 7 years time, what is the present worth, rebate being made at 5 per cent.? *Ans.* 245.

2dly, When P, T, R are given, to find S.

RULE.  $P \times r = S$ .

### EXAMPLES.

If a sum of money due 4 years hence produce 250 l. for the present payment, rebate being made at 6 per cent. what was the sum first due? *Ans.*  $250 \times 1.26247 = 315$  l. 12 s. 4 d.

If 245 l. be received for a debt payable 7 years hence, and an allowance of 5 per cent. to the debtor for present payment, what was the debt?

*Ans.* 344 l. 14 s. 9 d. 1.92 far.

3dly,

## 190 REBATE or DISCOUNT,

3dly, When S, P, R are given, to find T.

RULE.  $\frac{s}{p} = r$ , which being continually divided by R, till nothing remains, the number of those divisions will be equal to T.

### E X A M P L E S,

The present payment of 250 l. is made for a debt of 315 l. 12 s. 4 d. rebate at 6 *per cent.* in what time was the debt payable?

*Ans.*  $\frac{315.675}{250} = 1,26247$  which being divided continually, those divisions will be equal to 4, the number of years.

A person receives 245 l. now for a debt of 344 l. 14 s. 9 d. 1.92 far rebate being made at 5 *per cent.* I demand in what time the debt was payable? *Ans.* 7 years.

There is a debt of 441 l. 17 s. 3 d. 1.92 far. due at a certain time to come, but 6 *per cent.* being allowed to the debtor for the present payment of 350 l. I desire to know in what time the sum should have been made without any rebate? *Ans.* 4 years.

4thly, When S, P, T are given, to find R.

$\frac{s}{p} = r$  which being extracted by the rules of extraction, (the time given in the question shewing the power), will be equal to R.

E X-

## REBATE or DISCOUNT. 191

### EXAMPLES.

A debt of 315 l. 12 s. 4 d. 2 f. is due 4 years hence, but it is agreed to take 250 now, what is the rate *per cent.* that the rebate is made at ?

*Ans.*  $\frac{315.6715}{250} = 1.26247 : \sqrt[4]{1.26247} = 1.06 = 6 \text{ per cent.}$

The present worth of 344 l. 14 s. 9 d. 1.92 f. payable 7 years hence, is 245 l. at what rate *per cent.* is rebate made ? *Ans.* 5 per cent.

There is a debt of 441 l. 17 s. 3 d. 1.92 f. payable in 4 years time, but it is agreed to take 350 l. present payment ; I desire to know at what rate *per cent.* rebate is made at ? *Ans.* 6 per cent.

### MENSURATION.

**M**ensuration is threefold, either lineal, superficial, or solid.

1. Lineal, by some called running measure, and is taken by a line, and respects length without breadth. Cornice, freeze, cloth, &c. are thus measured.

2. Superficial, or flat square measure, is that which respects length and breadth.

3. Solid, or cube measure, which respects length, breadth, and depth, or thickness.

Mensuration has properly for its object all mathematical figures, whether comprehended under straight or curved lines ; and is performed, either  
by

by decimals, cross-multiplication, or duodecimals. But that we may treat of it with the greater precision and perspicuity, we shall divide it into four different kinds, *viz.* 1. Mathematical figures; 2. Mechanical mensuration. 3. Land-surveying; and, 4. Gauging. The first and two last will be performed by decimals, and mechanical mensuration by duodecimals; as this is not only the exactest and most minute method, but also the most expeditious, as will appear from one example performed three different ways, in a following page,

## MENSURATION of SUPERFICIES.

### SECT. I. CHAP. I. PROP. I.

To find the superficial content of a square, as fig. 1.

**RULE.** Multiply any one of the sides into itself, the product is the area.

What is the area of a square whose side is 12.5 inches?

$$\begin{array}{r}
 12.5 \\
 12.5 \\
 \hline
 625 \\
 250 \\
 125 \\
 \hline
 \end{array}$$

156.25 inches, or 1 foot  $12\frac{1}{4}$  inches square.

PROP.

P R O P. II.

What is the superficial content of a parallelogram, as fig. 2. whose length is 16.5 inches, and breadth 8.6 inches?

RULE. Multiply the length by the breadth, the product is the content.

$$\begin{array}{r}
 16.5 \\
 8.6 \\
 \hline
 \frac{1}{10} = 55 \\
 \frac{1}{10} = 55 \\
 1320 \\
 \hline
 143.9 \text{ inches.}
 \end{array}$$

P R O P. III.

What is the area of a rhombus, as fig. 3. whose perpendicular height is 16.5 inches, and one of its sides is 20.75 inches?

RULE. Multiply the perpendicular height into the side on which the perpendicular falls.

† R

20.75

$$\begin{array}{r}
 20.75 \\
 165 \\
 \hline
 10375 \\
 12450 \\
 3075 \\
 \hline
 142.375 \text{ inches.}
 \end{array}$$

N. B. The same rule serves a rhomboides, fig. 4.

### PROP. IV.

What is the superficial content of a plain triangle, as fig. 5. whose base is 10.8 inches, and perpendicular 14.6 inches?

RULE. Multiply the length of the base into  $\frac{1}{2}$  of the perpendicular, or *e contra*.

$$\begin{array}{r}
 10.8 \\
 7.3 \\
 \hline
 \frac{1}{2} = 36 \\
 756 \\
 \hline
 79.2 \text{ inches.}
 \end{array}$$

### PROP. V.

What is the superficial content of a trapezium, whose diagonal is 10.5 inches, and the sum of the two perpendiculars 6.6 inches, as fig. 6.?

RULE.

RULE. Multiply the diagonal into the half-sum of the two perpendiculars, or *e contra*.

$$\begin{array}{r} 10.5 \\ \frac{1}{2} \text{ sum} = 33 \\ \hline \frac{1}{3} = 35 \\ 315 \\ \hline 35 \bullet \end{array}$$

N. B. The same rule serves a parallelopleuron, whose figure is much the same, as both have 4 sides and 4 angles; but the first has its sides neither equal nor parallel, and the other has two opposite sides parallel, and the two angles at each end equal to one another.

# P R O P. VI

What is the superficial content of an irregular polygon, as fig. 7.?

RULE. Divide all such into triangles, and measure them by prop. 4.

Note, that all regular pentagons, hexagons, &c. as fig. 8. may be measured by drawing a perpendicular from the centre to any one of the sides, and then by multiplying the  $\frac{1}{2}$  of said perpendicular into the length of the side, and that product multiplied into the sum of the sides, will give the content required.

R 2

What

# 196 MENSURATION

What is the superficial content of fig. 8. whose perpendicular is 8.4 inches, and the side on which it falls is 12.8 inches?

$$\begin{array}{r}
 12.8 \\
 \frac{1}{2} \text{ perpend.} = 4.2 \\
 \hline
 256 \\
 512 \\
 \hline
 53.76 \\
 5 \\
 \hline
 268.80 \text{ inches.}
 \end{array}$$

## PROP. VII.

What is the superficial content of a circle, as fig. 9. whose diameter is 5.2 inches?

RULE. Multiply the square of the diameter by .7854, or the square of the periphery by .07957.

$$\begin{array}{r}
 5.2 \\
 5.2 \\
 \hline
 104 \\
 260 \\
 \hline
 27.04 \\
 .7854 \\
 \hline
 10816 \\
 13520 \\
 21632 \\
 18928 \\
 \hline
 21.237216
 \end{array}$$

Having

Having the diameter of a circle, to find the circumference.

1. Say,  $7 : 22 :: \text{diameter to the circumference.}$
2. Or  $113 : 355 :: \text{dia. : cir.}$
3. Or  $1 : 3.141593 :: \text{dia. : cir.}$

Having the circumference, to find the diameter.

1. Say  $1 : .318309 :: \text{circumference to the diameter.}$
2.  $355 : 113 :: \text{cir. : dia.}$
3.  $22 : 7 :: \text{cir. : dia.}$

### PROP. VIII.

What is the superficial content of an ellipsis, as fig. 10. whose transverse diameter is 36 inches, and conjugate diameter 16 inches?

RULE. Multiply the transverse and conjugate diameters into each other, and that product by .7854, and this last product will be the content required.

$$\begin{array}{r}
 36 \\
 16 \\
 \hline
 216 \\
 36 \\
 \hline
 576 \\
 .7854 \\
 \hline
 2304 \\
 2880 \\
 4608 \\
 4032 \\
 \hline
 452.3904 \text{ inches.} \\
 \hline
 R \ 3
 \end{array}$$

PROE.

## P R O P. IX.

What is the superficial content of a triangular prism, as fig 17. the length of the side of the two bases being 12 inches, and the height of the prism 30 inches.

**RULE.** Multiply the height by the length of the base's side, and that product by 3 for the area of all the sides; and then multiply the half of the side of the base by the perpendicular (suppose it to be 10.3 inches) let fall upon said side, for the area of one base; which being multiplied by 2 gives the area of both bases.

30	10.3
12	6
—————	—————
360	61 8
3	2
—————	—————

1080=area of all the sides. 123,6=areas of both bases.  
 123.6 area of both bases.

1203.6 square inches, the content required.

## P R O P. X.

What is the superficial content of a cylinder, as fig. 15. who length is 3 feet, and the diameter of the base 10 inches?

**RULE.** Multiply the circumference of the base by the length, and to that product add the area of the two bases, and you will have the content required.

$$\begin{array}{r}
 15.708 \\
 \underline{\phantom{00}5} \\
 78.540 \\
 \underline{\phantom{00}2} \\
 157.080 \text{ area of bases.}
 \end{array}$$

31.416 circumference of the base.  
36 length.

$$\begin{array}{r}
 188496 \\
 \underline{94248}
 \end{array}$$

1130.976 square inches.  
157.080

1288.056 square inch. the content required.

### PROP. XI.

What is the superficial content of a cone, as fig 16. whose diameter at the base is 16 inches, and length of the side 48 inches.

**RULE.** Multiply the circumference of the base into  $\frac{1}{3}$  of the length of the side, and the product

## 200. MENSURATION

duct is the area required, when you have added thereto the area of the base.

$$\begin{array}{r}
 16 \\
 16 \\
 \hline
 256 \\
 .7854 \\
 \hline
 1024 \\
 1280 \\
 2048 \\
 1792 \\
 \hline
 201.0624
 \end{array}$$

$$\begin{array}{r}
 3.1415 \\
 16 \\
 \hline
 188490 \\
 31415 \\
 \hline
 \end{array}$$

50.2640 = circumference of the base.  
 $24 = \frac{1}{2}$  of the length of the side of the cone.

$$\begin{array}{r}
 2010560 \\
 1005280 \\
 \hline
 \end{array}$$

1206.3360 = the area of the curve superficies;  
 201.0624 = the area of the base.

1407.3984 square inches, the content required.

### PROP. XII.

What is the superficial content of a triangular pyramid, as fig. 13. one side of whose base is 18 inches,

inches, the other 16 inches, and the third 20 inches, and the length of the pyramid's side 48 inches?

**RULE.** Take the sum of the triangular figures which constitute the pyramid, and thereto adding the area of the base, the sum is the area sought.

1st base=18	2d base=16	3d base=20
$\frac{1}{2}$ of the height=24	24	24
<hr/>	<hr/>	<hr/>
72	64	480
36	32	432
<hr/>	<hr/>	<hr/>
432	384	384
		1296=content of the three sides of the pyramid.

Then to find the area of the base, suppose the perpendicular let fall on the base=13.75, which multiplied by 10, the half of 20, gives the area. Thus, 13.75

$$\begin{array}{r} 13.75 \\ 10 \\ \hline 137.50 \\ 1296 \end{array}$$

1433.5 inches, the content of the whole pyramid.

What is the superficial content of a triangular pyramid, one side of whose base is 3 feet, the other 2 feet 8 inches, and third 3 feet 4 inches, and the height of the pyramid 8 feet, and the length of the perpendicular 2 feet  $3\frac{1}{2}$  inches, which falls upon the side, 40.

1ft

$$\begin{array}{r} \text{1st base } 36 \\ 48 \\ \hline \end{array}$$

$$288$$

$$144$$

$$\hline 1728$$

$$\begin{array}{r} \text{2d base } 32 \\ 48 \\ \hline \end{array}$$

$$256$$

$$128$$

$$\hline 1536$$

$$\begin{array}{r} \text{3d base } 40 \\ 48 \\ \hline \end{array}$$

$$1920$$

$$27.5$$

$$20$$

$$\hline 550.0$$

$$550$$

$$1920$$

$$1536$$

$$1728$$

$$\hline 5734 \text{ inches.}$$

## MENSURATION of SOLIDS.

### CHAP. II. PROP. I.

**W**Hat is the solid content of a cube, as fig. 1. whose side is 5.7 inches?

**RULE.** Multiply the side of the cube into itself, and that product again by the side.

$$5.7$$

$$5.7$$

$$\hline 399$$

$$285$$

$$\hline 32.49$$

$$5.7$$

$$\hline 22743$$

$$16245$$

$$\hline 185.193 \text{ solid inches.}$$

**PROP.**

P R O P. II.

What is the solid content of a parallelopipedon, as fig. 12. one side of the base being 9.5 inches, and the other side 7 inches, and the length thereof 20.3 inches?

RULE. Find the area of the end or base, and then multiply said area by the length.

$$\begin{array}{r}
 9.5 \\
 7 \\
 \hline
 66.5 \\
 20.3 \\
 \hline
 1995 \\
 13300 \\
 \hline
 1349.95
 \end{array}$$

The same figure 12 being a square prism, its content is found out by the same rule.

P R O P. III.

What is the solid content of a triangular prism, as fig. 17. whose base is 8.4 inches, height 6 inches, and length 22.7 inches?

RULE. Multiply the base by the height, and that product by the length.

$$\begin{array}{r}
 8.4 \\
 \underline{6} \\
 50.4 \\
 22.7 \\
 \hline
 3528 \\
 1008 \\
 1008 \\
 \hline
 1144.08
 \end{array}$$

## PROP. IV.

What is the solid content of a cylinder, as fig. 15. whose diameter at the end is 6.6 inches, and length 23 inches?

RULE. Square the diameter of the end, which multiply by .7854. and that product multiplied by the length, gives the content required.

$$\begin{array}{r}
 6.6 \\
 6.6 \\
 \hline
 \frac{1}{3} = 22 \\
 \frac{1}{3} = 22 \\
 400 \\
 \hline
 44.4 \\
 .7854 \\
 \hline
 1776 \\
 2220 \\
 3552 \\
 3008 \\
 \hline
 33.87176 \\
 23 \\
 \hline
 10161528 \\
 6674352 \\
 \hline
 769.05048
 \end{array}$$

PROP.

PROP. V.

What is the solid content of a square pyramid, as fig. 14. each of whose sides at the base is 3.6 inches, and height 12.75 inches?

RULE. Multiply the area of the base into  $\frac{1}{3}$  of its height?

$$\begin{array}{r}
 3.6 \\
 3.6 \\
 \hline
 \frac{1}{3} = 12 \\
 \frac{1}{3} = 12 \\
 110 \\
 \hline
 13.4 \\
 4.25 = \frac{1}{3} \text{ of the height.} \\
 \hline
 670 \\
 268 \\
 536 \\
 \hline
 56.950 \text{ solid inches.}
 \end{array}$$

PROP. VI.

To measure the frustum of a square pyramid, as fig. 14,

RULE. To the rectangle (or product) of the sides of the two bases, add  $\frac{1}{3}$  part of the square of their difference; that sum being multiplied into the frustum's height, will give its solidity, if the bases are square.

What is the solid content of the frustum of a square pyramid, the sides of whose two bases are 18 inches and 12 inches, and its height 18 feet?

$$\begin{array}{r}
 \dagger S \\
 18
 \end{array}$$

$\begin{array}{r} 18 \\ 12 \\ \hline 216 \end{array}$	$\begin{array}{r} 18 \\ 12 \\ \hline 6 \\ 6 \\ \hline 36 \\ \hline \frac{1}{3}=12 \end{array}$	$\begin{array}{l} 216 = \text{rectangle.} \\ 12 = \frac{1}{3} \text{ of the square.} \\ \hline 228 \\ 18 \\ \hline 1824 \\ 228 \\ \hline 4104 \\ 144)4104(28.5 \text{ feet solid.} \\ \hline 288 \\ \hline 1224 \\ 1152 \\ \hline 720 \\ 720 \\ \hline \end{array}$
---	--	---

Again, if it is a triangular pyramid, as fig. 13. the perimeter of whose greater base is 72 inches, and the perimeter of the lesser is 48 inches; *Quar.* the superficial content thereof, when the length is 18 feet?

**RULE.** Add both the perimeters together,  $\frac{1}{2}$  of which being multiplied by the length and divided by 12, to this quot add the two bases, and you have the content required.

$\begin{array}{r} 72 \\ 48 \\ \hline 120 \\ \frac{1}{2} = 60 \\ \hline 18 \end{array}$	$\begin{array}{l} \text{Suppose the greater base} = 2.25 \text{ feet,} \\ \text{and the lesser} = 1. \text{ feet.} \\ \hline 3.25 \end{array}$
$\begin{array}{r} 12)1080(90 \\ 108 \quad 3.25 \\ \hline \end{array}$	

0 93.25 feet, superficial content required.  
Again,

Again, let each side of the greater base be 25 inches, and each side of the lesser base be 9 inches, and the length 15 feet. what is the solid content ?

Note, that the same rule serves a square and triangular pyramid; but before you multiply by the length, multiply the sum by .433.

$$\begin{array}{r}
 25 \quad 25 \quad 134.4777 \\
 9 \quad 9 \quad 15 \\
 \hline
 225 \quad 16 \quad 6723885 \\
 \quad 16 \quad 1344777 \\
 \hline
 96 \quad 144 \quad 2017.1655 (14.007 \text{ solid feet}) \\
 16 \quad 144 \\
 \hline
 256 \quad 577 \\
 \hline
 \frac{1}{3} = 85.333 \quad 576 \\
 225 \quad 1116 \\
 \hline
 310.333 \quad 1008 \\
 .433 \quad 108 \\
 \hline
 \frac{1}{3} = 103444 \\
 1241333 \\
 \hline
 1344777
 \end{array}$$

P R O P. VII.

What is the solid content of a cone, as fig. 16: the area of whose circular base is 39.5 inches, and its height 11.55 inches ?

The rule is the same as for a pyramid ; thus,

$$\begin{array}{r} 39\ 5 \\ 3.85 = \frac{1}{4} \text{ of the height.} \\ \hline \end{array}$$

$$\begin{array}{r} 1975 \\ 3160 \\ 1185 \\ \hline 15207.5 \text{ solid inches.} \\ \hline \end{array}$$

To measure the frustum of a cone, the rule is the same as for the frustum of a square pyramid; only before you multiply by the height, multiply by .7854, the product whereof is the mean area.

### P R O P. VIII.

What is the solid content of a sphere, as fig. 17. whose periphery is 62.832 inches, and diameter 20 inches?

R U L E. Multiply the diameter into the circumference; and then multiply that product by  $\frac{1}{6}$  of said diameter.

$$\begin{array}{r} 62.832 \\ 20 \\ \hline 1256.640 \\ 3.333 = \frac{1}{6} \text{ of the axis being 20.} \\ \hline \frac{1}{3} = 418880 \\ 3769920 \\ \hline 4188.800 \text{ solid inches.} \\ \hline \end{array}$$

P R O P.

P R O P. IX.

What is the solid content of a spheroid, as fig. 18. the diameter of whose greatest circle is 6.5 inches, and the length 10 inches?

R U L E. Multiply the square of the diameter of the greatest circle by the length, then multiply that product by .5236.

$$\begin{array}{r}
 6.5 \\
 6.5 \\
 \hline
 325 \\
 390 \\
 \hline
 42.25 \\
 10 \\
 \hline
 422.50 \\
 .5236 \\
 \hline
 253500 \\
 126750 \\
 84500 \\
 211250 \\
 \hline
 \end{array}$$

221.221000 inches = the solidity of the spheroid.

P R O P. X.

What is the solid content of a parabolic spindle, as fig. 19. the square of whose greatest circle is 1296 inches, (whose root is 36), and its length 99 inches?

S 3

RULE?

**RULE.** Multiply the square of its greatest circle by .41888 being  $\frac{1}{3}$  of .7854, and that product by its length.

$\begin{array}{r} 36 \\ 36 \\ \hline 216 \\ 108 \\ \hline 1296 \\ .41888 \\ \hline 5184 \end{array}$	$\begin{array}{r} 542.88 \\ 99 \\ \hline 488592 \\ 488592 \\ \hline 1728)53745.12(31.10 \text{ solid feet.} \\ 5184 \\ \hline 1905 \\ 1728 \\ \hline 1771 \\ 1728 \\ \hline 432 \end{array}$
--	--

## MECHANICAL MENSURATION.

### S E C T. H. C H A P. I.

**W**E now proceed to the mensuration of the work of different artists by duodecimals, which are fractions of a foot, or of an inch, or of any part of an inch, having 12 for its denominator, and are written thus:

1 *feet. i. s. t. f.*  
       3   7   2   3   7   read thus, 3 feet 7 inches  
 2 seconds 3 thirds and 7 fourths.

NOTE,

# MENSURATION. 217

NOTE, 12 fourths = 1 third.  
 12 thirds = 1 second.  
 12 seconds = 1 inch.  
 12 inches = 1 foot.  
 3 feet = 1 yard.  
 6 yards = 1 rood lineal measure.

And as we will have occasion, in the sequel of  
 of this work, to reduce seconds, inches, and feet  
 to yards and roods;

Observe, that in

## *Superficial measure,*

144 seconds = 1 inch.  
 144 inches = 1 foot.  
 9 feet = 1 yard.  
 36 yards = 1 rood.

## *Solid measure.*

1728 seconds = 1 inch.  
 1728 inches = 1 foot.  
 27 feet = 1 yard.  
 216 yards = 1 rood.

Note, In Addition and Subtraction of duodeci-  
 mals we carry at, or borrow 12 in every column,  
 but that under feet.

In Multiplication of duodecimale, commonly  
 called Cross Multiplication,

Note,

Note, that Feet     $\times$ by feet give feet.  
                   feet     $\times$ by inches give inches.  
                   feet     $\times$ by seconds give seconds.  
                   inches  $\times$ by inches give seconds.  
                   inches  $\times$ by seconds give thirds.  
                   seconds  $\times$ by seconds give fourths.

PROP. I. OF GLASIERS WORK.

Glasiers commonly take their measure in inches and parts of an inch, and allow now-a-days 12 inches to the foot lineal, though of old they allowed no more than 8 inches.

What is the content of a pane of glass, whose length is 9 inches 3 seconds, and breadth 6 inches 6 seconds?

i.	f.		
9	3		
6	6		
<hr/>			
4	7	6	
	4	7	6
<hr/>			
Inches 5	0	1	6

Note, The above is a square pane; but suppose a glazier is obliged to cut his panes with an arch; then he is allowed measure from the top of the arch, on account of the loss of glass, and extraordinary labour.

PROP.

PROP. II.

What is the content of 4 arched panes of glass, whose length from the top of the arch to the bottom, is 10 inches 9 seconds, and breadth 7 inches 7 seconds?

$$\begin{array}{r}
 \text{I.} \quad \text{f.} \\
 10 \quad 9 \\
 7 \quad 7 \\
 \hline
 6 \quad 3 \quad 3 \quad 8 \\
 6 \quad 3 \quad 3 \\
 \hline
 6 \quad 9 \quad 6 \quad 3 \\
 4 \\
 \hline
 \text{Feet } 2 \quad 3 \quad 2 \quad 1
 \end{array}$$

PROP. III.

A gentleman employed a glazier for his house, in which there were 18 sash windows, each pane of

# 314 OF GLASIERS WORK.

of glafs was  $11\frac{1}{2}$  inches long and  $8\frac{1}{4}$  broad; what was the content of the whole?

$$\begin{array}{r}
 \text{I. f.} \\
 11 \quad 6 \\
 8 \quad 3 \\
 \hline
 7 \quad 8 \\
 \quad 2 \quad 10 \quad 6 \\
 \hline
 7 \quad 10 \quad 10 \quad 6 \\
 \hline
 18=6 \times 3 \\
 7 \quad 10 \quad 10 \quad 6 \quad 0 \\
 \hline
 23 \quad 8 \quad 7 \quad 6 \quad 0 \\
 \hline
 \text{Feet } 142 \quad 3 \quad 9 \quad 0 \quad \bullet
 \end{array}$$

12 panes in each window.

## PROP. IV.

Suppose a piece of glafs to be 7 feet 3 inches long, and 4 feet 7 inches broad; what is the superficial content thereof?

This

# OF GLASIERS WORK. 215

This question will be performed three different ways, in order to illustrate the excellency, minuteness, &c. of duodecimals, and how far they are preferable to every method hitherto invented.

Duodecimally,

*F. I.*

7 3

4 7

29

4 2 9

Feet 33 2 9 144)4785(33 : 2 : 9

432

465

432

33

12

396(2

288

108

12

1296(9

1296

Vulgarly,

87 inches.

55 d°

435

435

Decimally,

7.25

4.58333

$\frac{1}{3} = 241666$

5800

3625

2900

33.1291666

12

2.7500000

12

9.0000000

CHAP.

## CH A P. II. of PAINTERS and JOINERS WORK.

**W**E shall treat of these two under the same head, as the dimensions are both taken the same way, viz. generally in feet, and always reduced to square yards: and you will observe, that the room or piece of work is measured, by taking a line, and applying one end thereof to any corner of the room, going into every corner with the line, till you come to the place where you began; and then find how many feet and inches the line contains, and set it down for the compass, and then apply said line to the top of the cornice (indenting the line where-ever the plane or brush goes), measure over the mouldings till you come to the floor for the height; which when multiplied by the compass, its product will give the answer in square feet, which divided by 9 gives yards.

### P R O P. I.

What is the content in square yards of a painted room, whose compass is 45 feet 8 inches, and height 10 feet 6 inches?

F.	i.
45	8
10	6
<hr/>	
456	8
22	10
<hr/>	
9)479	6
<hr/>	

Square yards    53   2   5

P R O P.

## PROP. II.

If the height of a painted room be 12 feet 4 inches, and the compass 84 feet 11 inches, how many square yards will it contain?

$$\begin{array}{r}
 \text{F. i.} \\
 84 \text{ } 11 \\
 12 \text{ } 4 \\
 \hline
 1019 \\
 28 \text{ } 3 \text{ } 8 \\
 \hline
 9) 1047 \text{ } 3 \text{ } 8 \\
 \hline
 \text{Yards } 116 \text{ } 3 \text{ } 3 \text{ } 8
 \end{array}$$

N. B. Double work is allowed in window-shutters, and the outside of the doors must be measured and added, before you bring them to square yards.

## PROP. III

What is the superficial content of a piece of wainscoting, that is 9 feet 3 inches long, and 6 feet 6 inches broad?

$$\begin{array}{r}
 \text{F. i.} \\
 9 \text{ } 3 \\
 6 \text{ } 6 \\
 \hline
 55 \text{ } 6 \\
 4 \text{ } 7 \text{ } 6 \\
 \hline
 9) 60 \text{ } 1 \text{ } 6 \\
 \hline
 \text{Yards } 6 \text{ } 6 \text{ } 1 \text{ } 6 \\
 \text{† T} \qquad \text{PROP.}
 \end{array}$$

## PROP. IV.

There is a room wainscotted, the compass of which is 47 feet 3 inches, and height 7 feet 6 inches; how many yards are therein?

F.	i.	
47	3	
7	6	
33 <sup>0</sup>	9	
23	7	6
		yards.
9)	354	4 6 = 39 : 3 : 4 : 6

## PROP. V.

There is a room of wainscot 129 feet 6 inches in circumference, and 16 feet 9 inches high, (being girt over the mouldings); there are 2 windows, each 7 feet 3 inches high, and the breadth of each from cheek to cheek 5 feet 6 inches; the breadth of the shutters each 4 feet 6 inches; the cheek boards, top and bottom D<sup>o</sup>, of each window, taken together, are 24 feet 6 inches, and their breadth 1 foot 9 inches; the door-case 7 feet high, and 3 feet 6 inches wide; the door 3 feet 3 inches wide; I demand how many square yards are contained in said room?

N. B. The door and shutters are at work and half.

F.

<p><i>F. i.</i>  129 6  16 9  <hr/> 2072  97 1 6  <hr/> 2169 1 6 content of  the rooms.</p>	<p><i>F. i.</i>  7 3  4 6  <hr/> 29  3 7 6  <hr/> 32 7 6  16 3 9  <hr/> 48 11 3  2  <hr/> 97 10 6 shutters at work  and half.</p>
---	---

<p><i>F. i.</i>  33  7  <hr/> 22 9  11 4 6  <hr/> 34 1 6 door at  work and half</p>	<p><i>F. i.</i>  24 6  1 9  <hr/> 24 6  18 4 6  <hr/> 42 10 6  2  <hr/> 85 9 cheek-boards, &amp;c.</p>
---	--

# 220 OF PAINTERS WORK.

F. i.	F. i.
7 3	3 6
5 7	7
<hr/>	<hr/>
36 3	24 6 door-case.
3 7 6	+79 9 two windows.
<hr/>	<hr/>
39 10 6	104 3 to be deducted.
2	
<hr/>	
79 9 two windows.	

Content of the room	2169 1 6
Shutters at work and half	97 10 6
Door at D <sup>o</sup>	34 1 6
Check-boards, &c.	85 9
	<hr/>
	2386 10 6
Deduct window-lights, &c.	104 3
	<hr/>
	9) 2282 7 6
	<hr/>
Square yards	253 5 7 6

## PROP. VI.

What is the superficial content of a floor, whose length is 25 feet 7 inches, and breadth 17 feet 5 inches?

N. B. Flooring may be computed either by the square foot or square yard, according to the agreement made betwixt the parties contracting.

F.

$$\begin{array}{r}
 \text{F. } i. \\
 25 \quad 7 \\
 17 \quad 5 \\
 \hline
 434 \quad 11 \\
 10 \quad 7 \quad 11 \\
 \hline
 \text{Square feet } 445 \quad 6 \quad 11 \\
 \hline
 \end{array}$$

P R O P. VII.

In a floor 49 feet 7 inches 4 seconds long, and 26 feet 6 inches broad, how many square yards ?

$$\begin{array}{r}
 \text{F. } i. \quad s. \\
 49 \quad 7 \quad 4 \\
 26 \quad 6 \\
 \hline
 1274 \\
 15 \quad 10 \quad 8 \\
 24 \quad 9 \quad 8 \\
 \hline
 9)1314 \quad 8 \quad 4 \\
 \hline
 \text{Square yards } 146 \quad 0 \quad 8 \quad 4 \\
 \hline
 \end{array}$$

CHAP. III. of SAWYERS WORK.

**S**AWYERS measure by the superficial foot, and are generally paid so much per 100 feet; they account the depth of the kerf for the breadth, and the length for the length; and having found the content of one kerf, multiply it by the number

of kerfs of the same dimension, and you will have the number of feet in them all.

## P R O P. I.

There is a log of timber whose breadth is 1 foot 7 inches, and length 23 feet 9 inches, and there are 16 kerfs in said log; how many feet does it contain in whole?

$$\begin{array}{r}
 \text{F. i.} \\
 23 \quad 9 \\
 1 \quad 7 \\
 \hline
 23 \quad 9 \\
 13 \quad 10 \quad 3 \\
 \hline
 37 \quad 7 \quad 3 \\
 \text{Numb. of kerfs } 16 = 8 \times 2 \quad 2 \\
 \hline
 75 \quad 2 \quad 6 \\
 8 \\
 \hline
 1,00)6,018 \\
 \hline
 \text{Hundred } 6 \quad 01 \quad 8 \\
 \hline
 \end{array}$$

Note, If the kerf be but 6 inches deep or less, in that case sawyers have a custom, in some places, to be paid for kerf and half, i. e. for half so much more than it comes to by measure; and this they insist upon for extraordinary trouble in fitting, removing, and new-binding their timber.

P R O P.

P R O P. II.

In a log of timber 18 feet 8 inches long, and 5 inches broad, there are 9 kerfs; how many superficial feet are in said log?

$$\begin{array}{r}
 \text{F.} \quad \text{i.} \\
 18 \quad 8 \\
 \quad \quad 5 \\
 \hline
 7 \quad 9 \quad 4 \text{ work} \\
 3 \quad 10 \quad 8 \quad \text{and half.} \\
 \hline
 11 \quad 8 \quad \quad \times 9 = 105 \text{ sq. feet.}
 \end{array}$$

C H A P. IV. Of SLATERS WORK.

**S**LATERS measure their work either by feet and inches, or yards and feet, and always reduce it to square roods: and observe, that in measuring the roof, you take the length from the skews on the one gable to those on the other; and for the breadth, measure down from the top of the ridging as far as the slates come over the side-wall, and they are always allowed 9 inches for the eave; which must be added to the breadth before you multiply it into the length; and lastly measure the thickness of the base of the chimney on each side of the roof, and take the length of it to the ridging; which product must be deducted from the total content, before you reduce it to roods. Some insist, that this kind of work ought to be measured by the Scotch ell of 37 inches.

P R O P.



$$\begin{array}{r}
 \text{F. } i. \\
 28 \quad 8 \\
 8 \quad 5 = \frac{1}{2} \text{ perpendicular} \\
 \hline
 229 \quad 4 \\
 11 \quad 11 \quad 4 \\
 \hline
 241 \quad 3 \quad 4 \\
 \hline
 9)965 \quad 1 \quad 4 \\
 \hline
 36)107 \quad 2 \quad 1 \quad 4 \\
 \hline
 \text{Roods } 2 \quad 35 \quad 2 \quad 1 \quad 4
 \end{array}$$

# C H A P. V. of MASONS WORK.

**M**ENSURATION of masons work is two-fold; and it consists either, 1. of hewn work, or, 2. of laid or built work.

Hewn work is measured by the square foot, and the dimensions taken in feet and inches; and the hewn work of doors and windows ought to be measured into the chack.

All laid work is measured by the square rood of 36 square ells, when the dimensions are taken in Scots ells and inches; but if you take the dimensions in feet, divide by 9.5 for the content in Scots ells; and this method is warmly contended for by some, while others insist that the dimensions may be taken in yards or feet, and reduced as flaters work: in the first case, the employer is a gainer, and by the last he is a loser.

We

We could wish that, in measuring masons work, regard was paid to the following observations, which may be of use and advantage to every practical measurer.

*Observation 1.* In measuring vents, (where they are not paid for by the piece, which is the common practice and safest method), after taking the circumference at the top and bottom of the vent, find the mean circumference; which multiplied by the perpendicular height, will give the content.

*Obs. 2.* That the standard thickness of walls is 2 feet, consequently if a wall is 3 feet thick, the content of that part will be  $\frac{1}{2}$  more; or if any part of said wall be 20 inches thick,  $\frac{1}{3}$  must be subtracted from the content, before it is reduced to rods.

*Obs. 3.* That the side-walls ought to be measured without, and the gables within; yet some masons insist that the girth of the whole house should be taken on the outside, and a few practical measurers judge this neither unreasonable nor unjust.

*Obs. 4.* That the height of side-walls is to be taken from the lowest part of the foundation to the top of the wall, and  $\frac{1}{2}$  of the thickness of the wall at the top is to be added thereto, on account of the extraordinary trouble of levelling the same, the employer furnishing materials, and the mason only agreeing for workmanship.

*Obs. 5.* That in measuring the gable above side-wall height, the breadth thereof ought to be taken at the top of the side-wall, and also at the base of the chimney-stalk, by which you find the mean proportional; but the common way (though erroneous) is to take the half of these two when added for the mean breadth, which multiplied into the perpendicular let fall from the base of the chimney.

chimney-stalk down to the side-wall height, gives the content thereof; and the thickness must be taken in three or four places, so that the mean thickness may be ascertained.

*Obs.* 6. That the chimney head or stalk must be measured by taking the breadth and one of the ends, for the whole breadth; which being multiplied into the height, will give the content thereof; but in measuring the hewn work of the same, the girth of the whole must be multiplied into the height for the content.

# P R O P. I.

There is a vent whose girth at the top is 4 feet, and at the bottom 7 feet, and its height 36 feet;  
*Quer.* the content ?

True method thus.

F.	36	
7	5.291 mean proportional.	
4	<hr/>	
<hr/>	36	
28) 5.291	324	
25	372	
<hr/>	180	
102) 300	<hr/>	
204	190.476 feet.	
<hr/>		
1049) 9600		
9441		
<hr/>		
10581) 15900		
10581		
<hr/>		
5319		

False

False method thus.

F.

7

4

—

2)11

—

5

6 mean breadth.

36

—

198 feet.

## P R O P. II.

There is a house 30 feet long and 20 feet wide within the walls, and the height of the side-wall is 36 feet; and the thickness of the gable above side-wall height (being taken in 3 different places) the mean breadth is found to be 18 inches, but below side-wall height is 3 feet thick, on account of the vents, and the breadth of the gables at side-wall height 24 feet, and at the base of the chimney 8 feet, and perpendicular height 14 feet; in this house there are 12 windows, whose height is 4 feet 6 inches, and the length of the lintels and soles 3 feet 3 inches, and the breadth of the hewn work from the outside into the chacks 10 inches; and there are 8 hearth-stones 3 feet 6 inches long, and 1 foot 10 inches broad; and also 8 chimney pieces 3 feet 8 inches high, and breadth of the hewn work 22 inches; and lintels 5 feet 6 inches long, and breadth of the hewn work 11 inches; the girth of the 2 chimney stalks 22 feet 4 inches each, and height 8 feet 8 inches; I demand the number of roods of built work, and how many feet of hewn work are in said house?

F.

*F.*

36 as 6 feet are added for the thickness of the gables.

37 as 1 foot is added for levelling the top of the

— side-wall.

252

108

---

1332

2

---

2664=content of both side-walls.

*F.*

20 the wideness of the gable within.

36

---

720

2

---

1440 to which its  $\frac{1}{2}$  being, added because it is 3 feet  
720 thick.

---

2160=content of both gables side wall height.

† U

*F.*

# 230 OF MASONS WORK.

F.

13.856 the mean proportional of the gables,

14

55424

13856

193.984

2

387.968  $\frac{1}{4}$  must be deduced, because the gable is  
96992 only 18 inches of mean breadth.

290.976=29 of 11 inches=content of 2 gables.

12

11.712

F.

10 the breadth of the chimney-stalk and 1 end,  
8 8 the perpendicular height.

86 8

2

173 4=content of two chimney-stalks.

F.

Windows  $\begin{array}{r} F. \quad i. \\ 4 \quad 6 \text{ high.} \\ \hline 10 \end{array}$

$\begin{array}{r} 3 \quad 9 \\ \hline 2 \end{array}$

$\begin{array}{r} 7 \quad 6 \\ \hline 12 \end{array}$

$\begin{array}{r} 90 \quad 0 \\ 65 \quad 0 \text{ lintels and soles.} \\ \hline \end{array}$

Feet 155 0=content of 12 windows.

Lintels  $\begin{array}{r} F. \quad i. \\ 3 \quad 3 \\ \hline 10 \end{array}$

Soles D<sup>o</sup>  $\begin{array}{r} 2 \quad 8 \quad 6 \\ \hline 2 \quad 8 \quad 6 \end{array}$

$\begin{array}{r} 5 \quad 5 \\ \hline 12 \end{array}$

$\begin{array}{r} 65 \quad 0 \end{array}$

Hearths.  $\begin{array}{r} F. \quad i. \\ 3 \quad 6 \\ \hline 1 \quad 10 \end{array}$

$\begin{array}{r} 3 \quad 6 \\ \hline 2 \quad 11 \end{array}$

$\begin{array}{r} 6 \quad 5 \\ \hline 8 \end{array}$

51 4=content of 8 hearths.

# 232 OF MASONS WORK.

	<i>F.</i>
Content of 2 side-walls	= 2664
D° of 2 gables side-wall high	= 2160
D° of D° above side-wall height	= 290 11
D° of two chimney-stalks.	= 173 4

9)5288 3

36=6X6 & 6)587 5 3

6)97 5

Roods 16 11 5 3

	<i>F.</i>	<i>i.</i>
Chimneys	3	8
	1	10
	3	8
	3	0 8
	6	8 8
		2
	13	5 4

Lintels of D°	<i>F.</i>	<i>i.</i>
	5	6
		11
	5	0 6
	13	5 4
	18	5 10
		8

Content of eight chimneys=147 10 8

*F.*

	F.	i.
Chimney-stalks.	22	4
	8	8
	<hr/>	
	178	8
	14	10 8
	<hr/>	
	193	6 8
		2
	<hr/>	
Content of the above=	387	2 4
	<hr/>	

	F.	i.
12 Windows	= 155	0
8 Chimneys	= 147	10 8
8 Hearths	= 51	4
2 Chimney stalks=	387	1 4
	<hr/>	

Feet 741 4 0 = all the hewn work.

## CHAP. VI. OF SOLIDS.

### PROP. I.

**T**Here is a stone whose length is 16 feet 4 inches, its breadth 9 feet 6 inches, and its thickness 6 feet 8 inches; what is the solid content of said stone?

U 3

F.

F. i.

16 4

9 6

---

147

8 2

---

155 2

6 8

---

931

103 5 4

---

Solid feet 1034 5 4

---

## P R O P. II.

In a piece of timber, whose length is 17 feet 6 inches, breadth 1 foot 11 inches, and thickness 2 feet 7 inches, how many solid feet?

F. i.

17 6

1 11

---

17 6

16 0 6

---

33 6 6

2 7

---

67 1

19 6 9 6

---

86 7 9 6

---

P R O P.

PROP. III.

How many solid yards of digging are in a cellar, that is 8 yards 1 foot 4 inches long, 5 yards 8 inches broad, and 2 yards 1 foot 4 inches deep?

	<i>P.</i>	<i>i.</i>
8 Yards 1 foot	25	4
5 D°	15	8
	380	
	16	10 8
	396 10 8	
2 D° 1 foot	7	4
	2778 2 8	
	132	3 6 8
	27) 2910 6 2 8	
Solid yards	107 21 6 2 8	

We shall now treat of the mensuration of round timber whose bases are equal; which kind of mensuration has been overlooked by a great number of authors, whilst others have handled it after a most erroneous method; as their usual way to measure round timber trees, is to girt them about  
the

the middle with a line, and then to take  $\frac{1}{4}$  of said girth for the side of a square; by which method they measure a piece of timber as if it was an exact square: others taking the circumference with a line, find the content of the circle, and to bring it to a square, throw away  $\frac{1}{4}$  of the content before they multiply it by the length; and lastly some throw away  $\frac{1}{5}$  of the content. I confess I have little hopes of opening the eyes of such as have adopted either of the above erroneous methods; however, those that are unprejudiced, I expect will listen to truth: therefore that all these methods are erroneous, I shall make appear as follows. If the circumference of a circle be 1, the area will be .07958; then the 4th part of 1 is 25. which being squared makes .0625; and this they take for a mean area, instead of .07958; therefore the true content always bears such proportion to the content found by the false way, as .07958 to .0625, which is nearly as 23 to 18; so that by this method, as well as the other two mentioned above, more than  $\frac{1}{5}$  is lost of what the true content ought to be. The different errors will appear by the following examples.

#### P R O P. IV.

If a piece of timber be 96 inches in circumference, and 18 feet long, how many solid feet will be contained therein?

The

The true way thus,

$$\begin{array}{r}
 96 \\
 96 \\
 \hline
 576 \\
 864 \\
 \hline
 9216 \\
 .07958 \\
 \hline
 73728 \\
 46080 \\
 82944 \\
 64512 \\
 \hline
 733.40928 \\
 18 \\
 \hline
 586727424 \\
 73340928 \\
 \hline
 144) 13201.36704 (91 \text{ feet,} \\
 1296 \\
 \hline
 241
 \end{array}$$

The false way thus,

$$\begin{array}{r}
 4) 96 \\
 \hline
 24 \\
 24 \\
 \hline
 96 \\
 48 \\
 \hline
 576 \\
 18 \\
 \hline
 4608 \\
 576 \\
 \hline
 144) 10368 (72 \text{ feet,} \\
 1008 \\
 \hline
 288 \\
 288 \\
 \hline
 00
 \end{array}$$

PROP. V:

If a log of timber be 86 inches in girth, and 20 feet long; *Quar.* the content?

The

The true way thus,

$$\begin{array}{r}
 86 \\
 86 \\
 \hline
 516 \\
 688 \\
 \hline
 7396 \\
 .07958
 \end{array}$$

$$\begin{array}{r}
 59168 \\
 36950 \\
 66564 \\
 51772 \\
 \hline
 588.57368
 \end{array}$$

$$\begin{array}{r}
 20 \\
 \hline
 \text{feet}
 \end{array}$$

$$144) 11771.4736 (81.7460$$

$$1152$$

$$251$$

$$144$$

$$1074$$

$$1008$$

$$667$$

$$576$$

$$913$$

$$864$$

$$49$$

The 2d false way.

$$43 = \frac{1}{2} \text{ of the circumf.}$$

$$138 = \frac{1}{2} \text{ of the diam.}$$

$$\begin{array}{r}
 4) 587 \ 8 \\
 146 \ 11 \\
 \hline
 440 \ 9 \\
 20
 \end{array}$$

$$144) 8815 (61.215 \text{ feet.}$$

$$864$$

$$175$$

$$144$$

$$310$$

$$288$$

$$220$$

$$144$$

$$760$$

$$720$$

$$40$$

N. B.

N. B. If proprietors of woods or cut round timber do agree to sell said woods or round timber as if exactly squared, then I allow that  $\frac{1}{4}$  of the girth at the middle may be multiplied into itself, and that product by the length, or  $\frac{1}{4}$  of the content of the mean circle may be deducted from the whole content, before it is multiplied by the length: but in either cases the proprietor is a considerable loser, as is shewn above.

Of round timber whose bases are unequal, *i. e.* having the one end greater than the other;

The usual way is, to take a fourth part of the girth in the middle of the piece; but if this way was erroneous in the former case, (as I have already shewn), it will be still more so in the present; and the more tapering the tree is, the greater will the error be; for all such timber ought to be considered as the frustum of a cone, and should be measured accordingly, as in chap. 2. prop. 7. of solids; and thus you will find the true content.

### P R O P. VI.

If a piece of timber be 9 inches diameter at the lesser end, and 36 inches at the greater, and 24 feet long, what is the solid content thereof?

$$\begin{array}{r} 36 \\ 9 \\ \hline 324 \text{ rectangle.} \end{array} \quad \begin{array}{r} 36 \\ 9 \\ \hline 27 \text{ difference.} \end{array}$$

$$\begin{array}{r} 189 \\ 54 \\ \hline 3)729 \text{ the square of the difference.} \end{array}$$

$$\begin{array}{r} 243 \\ 324 \text{ rectangle added.} \\ \hline \end{array}$$

$$\begin{array}{r} 567 \\ .7854 \\ \hline \end{array}$$

$$\begin{array}{r} \text{A mean area } 445.3318 \\ 24 \\ \hline \end{array}$$

$$\begin{array}{r} 144)10687.7232(74.22 \\ 1008 \\ \hline \end{array}$$

$$\begin{array}{r} 607 \\ 576 \\ \hline \end{array}$$

$$\begin{array}{r} 317 \\ 288 \\ \hline \end{array}$$

$$\begin{array}{r} 292 \\ 288 \\ \hline \end{array}$$

$$4$$

PROP. VII.

If a piece of timber be 136 inches in circumference at one end, and 32 inches d<sup>o</sup> at the other, and 21 feet long, how many solid feet of timber are contained therein?

$$\begin{array}{r} 136 \\ 32 \\ \hline 272 \\ 408 \\ \hline 4352 \end{array}$$

$$\begin{array}{r} 136 \\ 32 \\ \hline 104 \text{ difference.} \\ 104 \\ \hline 416 \\ 1040 \end{array}$$

3) 10816 the sq. of the difference.

$$\begin{array}{r} 3605.333 \\ 4352 \text{ rectangle added.} \\ \hline \end{array}$$

$$\begin{array}{r} 7957.333 \\ .7958 \\ \hline \end{array}$$

$$\begin{array}{r} 63658666 \\ 397866666 \\ 7161600000 \\ 55701333333 \\ \hline 633.24458666 \\ 21 \end{array}$$

$$\begin{array}{r} 63324458666 \\ 1266489173333 \\ \hline \end{array}$$

$$144) 13298.13632 \quad (92.348$$

$$1296$$

$$\begin{array}{r} 338 \\ 288 \\ \hline \end{array}$$

$$\begin{array}{r} 501 \\ 432 \\ \hline \end{array}$$

$$\begin{array}{r} 693 \\ 576 \\ \hline \end{array}$$

$$\begin{array}{r} 1176 \\ 1152 \\ \hline \end{array}$$

$$24$$

† X

Duodecimally

Duodecimally thus,

$$\begin{array}{r}
 F. \quad i. \\
 11 \quad 4 \\
 2 \quad 8 \\
 \hline
 22 \quad 8 \\
 7 \quad 6 \quad 8 \\
 \hline
 30 \quad 2 \quad 8
 \end{array}$$

$$\begin{array}{r}
 F. \quad i. \\
 8 \quad 8 \text{ differ.} \\
 8 \quad 8 \\
 \hline
 69 \quad 4 \\
 5 \quad 9 \quad 4 \\
 \hline
 3)75 \quad 1 \quad 4 \\
 \hline
 25 \quad 0 \quad 5 \quad 4 \\
 30 \quad 2 \quad 8 \\
 \hline
 \end{array}$$

$$88 : 7 :: 55 \quad 3 \quad 1 \quad 4$$

$$88 = 11 \times 8 \quad 11)386 \quad 9 \quad 9 \quad 4$$

$$8)35 \quad 1 \quad 11 \quad 9$$

$$21 = 7 \times 3$$

$$\begin{array}{r}
 4 \quad 4 \quad 8 \quad 11 \\
 7 \\
 \hline
 30 \quad 9 \quad 2 \quad 5 \\
 3
 \end{array}$$

$$\text{Solid feet } 92 \quad 3 \quad 7 \quad 3$$

Of squared timber, *i. e.* such as have equal bases, and the sides straight and parallel;

**RULE.** Multiply the breadth by the thickness, and that product multiplied by the length, will give the solid content.

## P R O P. VIII.

If a piece of timber is 1 foot 3 inches square, and 18 feet long, *Quar.* the content?

$$\begin{array}{r}
 15 \\
 15 \\
 \hline
 225 \\
 18 \\
 \hline
 144)4050(28.125 \\
 288 \\
 \hline
 1170 \\
 1152 \\
 \hline
 180 \\
 144 \\
 \hline
 360 \\
 288 \\
 \hline
 720 \\
 720 \\
 \hline
 \end{array}$$

Or thus,

	<i>F.</i>	<i>i.</i>
	1	3
	1	3
	1	3
	3	9
	1	6 9
18=6×3		6
	9	4 6
		3
Feet	28	6

Of unequal squared timber, *i. e.* such timber as has the one end thicker than the other, and this is the case with most trees when hewn, and brought to their squares; the common method is, to take a girth with a line about the middle of the piece of timber, for a mean square: but this in most cases is very erroneous, especially when there is a great disproportion between the ends; therefore all such solids, being the frustums of pyramids, ought to be measured according to Prop 6. of solids. We shall give an example both ways, so that the error may be the more obvious.

P R O P. IX.

If a piece of timber be 25 inches square at the  
X 2 greater

greater end, and 9 inches square at the lesser end, and 20 feet long; how many solid feet of timber are in said tree?

The true way thus,

$$\begin{array}{r} 25 \quad 25 \\ 9 \quad 9 \\ \hline 225 \quad 16 \text{ difference.} \\ 16 \\ \hline \end{array}$$

$$\begin{array}{r} 96 \\ 16 \\ \hline 3)256 \\ \hline 85.333 \\ 225 \\ \hline \end{array}$$

$$\begin{array}{r} 310.333 \\ 20 \\ \hline \end{array}$$

144)6206.660)43.101 solid feet.

$$\begin{array}{r} 576 \\ \hline 446 \\ 432 \\ \hline 146 \\ 144 \\ \hline 260 \\ 144 \\ \hline 116 \end{array}$$

The false way thus,

$$\begin{array}{r} 25 \\ 9 \\ \hline 2)34 \\ \hline \end{array}$$

17 the side of  
17 the square in  
the middle.

$$\begin{array}{r} 119 \\ 17 \\ \hline \end{array}$$

$$\begin{array}{r} 289 \\ 20 \\ \hline \end{array}$$

$$\begin{array}{r} 144)5780)40.138 \\ 576 \\ \hline \end{array}$$

$$\begin{array}{r} 200 \\ 144 \\ \hline \end{array}$$

$$\begin{array}{r} 560 \\ 432 \\ \hline \end{array}$$

$$\begin{array}{r} 1280 \\ 1152 \\ \hline \end{array}$$

$$\begin{array}{r} 128 \end{array}$$

P R O P.

PROP. X.

If a piece of timber be 32 inches broad, and 20 inches deep at the greater end, and 10 inches broad and 6 inches deep at the lesser end, and 18 feet long; *Quar.* the solid content thereof?

The false way thus,	
Breadth.	Depth.
32	20
10	6
<hr/>	<hr/>
2)42	2)26
<hr/>	<hr/>
21	13
13	
<hr/>	
273	
18	
<hr/>	

144)4914(34.125 solid, false.

X 3

The

The true way thus,

$$\begin{array}{r} 32 \\ 20 \\ \hline 640 \\ 60 \end{array} \quad \begin{array}{r} 10 \\ 6 \\ \hline 60 \end{array}$$

$\dot{3}8400$  (195.959 mean proportional.

I 640 = the greater base.

--- 60 = the lesser do.

$$\begin{array}{r} 29)284 \\ 261 \\ \hline \end{array} \quad \begin{array}{r} 895.959 \end{array}$$

$$\begin{array}{r} 385)2300 \\ 1925 \\ \hline \end{array}$$

$$\begin{array}{r} 3909)37500 \\ 35181 \\ \hline \end{array}$$

895.959 the sum  
6  $\frac{1}{3}$  of the length

$$\begin{array}{r} 39185)231900 \\ 195925 \\ \hline \end{array} \quad \begin{array}{r} 144)5375.754(37.331 \\ 432 \\ \hline \end{array}$$

$$\begin{array}{r} 391909)3597500 \\ 3527181 \\ \hline \end{array} \quad \begin{array}{r} 1055 \\ 1008 \\ \hline \end{array}$$

$$\begin{array}{r} 70319 \\ 477 \\ 432 \\ \hline \end{array}$$

$$\begin{array}{r} 455 \\ 432 \\ \hline 234 \\ 144 \\ \hline 90 \end{array}$$

P R O P.

P R O P. XI.

To find the content of any irregular body, such as the root of a tree or gooseberry bush, &c.

RULE. Immerse the said body into water in a parallelopiped, having before-hand measured the height of the water, and then after the immersion find the solidity of the water raised, and you will have the solidity of the body immersed.

For example, if the parallelopiped be 6 feet long and 4 feet broad, and the water is raised 3 feet, the body will be 72 feet solid.

LAND-SURVEYING.

S E C T. III. C H A P. I.

**A**S we have inserted a table of land-measure, page 32. and also some other observations necessary to be attended to, it would be useless to resume either in this place: however, before we proceed to the examples adapted to explain this part of mensuration, it may be of use to insert some geometrical problems, which will enable the learner to protract any field upon paper.

P R O B L E M I.

To draw a perpendicular to a given line, thro' a given point: In doing this three cases may happen; for the given point may either be in the given line, or at one of the extremities of the given line, or out of the given line. 1st, then, if the point C, fig. 21. be given in the line A B, to draw a perpendicular through this point C; take at  
pleasure

pleasure from the given point C, upon the given line A B on both sides, the two equal distances C D, C E; and describe from the points D, E, with any opening of the compasses greater than C D, or C E, two arcs of a circle on both sides, which intersect here at the points F, G, through which you must draw the right line F G, which will pass through the given point C, and will be perpendicular to the given line A B. Q. E. F.

Secondly, If the point through which you are to draw a perpendicular to the line A B, as fig. 22. is given, in one of its extremities, as A, describe at pleasure, from this point A, the arc of a circle C D E, and, with the same opening of the compasses, set off twice from the point C, where it cuts the line A B in D, and from D in E, describe from the two points E, D, still with the same opening of the compasses, two arcs of a circle, which will cut here in the point F; thro' which, and through the given point A, draw the right line A F, which will be perpendicular to A B. Q. E. F.

Lastly, if the point through which you are to draw the perpendicular, be given out of the given line A B, as C, fig. 23. describe at pleasure, thro' the point C, the arc of a circle E D, which cuts the given line A B in two points, as D E; from which describe, with the same opening of the compasses, two arcs of a circle, and draw through their intersection F, and the given point C, the right line C F, which will be the perpendicular required. Q. E. F.

## P R O B. II.

To draw a regular square upon a given right line A B, fig. 1. draw the line A D perpendicular, and

and equal to the line A B, and describe an arc of a circle from the point D, with the extent A B or A D, and with the same extent describe from the point B another arc cutting the first in the point F, through which draw the right lines F B, F D.  
Q. E. F.

### P R O B. III.

To make an angle of any given magnitude, suppose 70 degrees.

Upon the given line A B. fig. 24. draw an arc of a circle with the compasses opened to the extent of 60 degrees from the point B, which will cut the given line A B at E, from which point E cut off 70 degrees (with the compasses opened to that extent) from said arc at the point F, through which, and the point A, draw the right line A D.  
Q. E. F.

### P R O B. IV.

To inscribe a regular polygon in a given circle.

First, If you would inscribe a hexagon in a given circle whose centre is A, the radius being set off with the compasses on the circumference, will go round six times exactly; and thus you have the sides of the hexagon. Q. E. F. But if you would inscribe any other regular polygon, for example a pentagon, you must, on the centre A, make the angle B A C, equal to the angle at the centre, which in a pentagon is 72 degrees, and the cord B C will be the side of the pentagon.

The angle at the centre is found by dividing 360 degrees, by the number of the sides of the polygon

gon to be inscribed, as by 7 for a heptagon, 8 for an octagon, and so on.

P R O P. I.

What is the content of a field exactly square, as fig. 1. each of whose sides is 8 chains 70 links?

Having measured the diagonal of said field, or any similar one upon the ground, and likewise the four sides, it will be easy to protract it upon paper; thus draw a line exactly of the same length with that diagonal you measured with the chain on the ground; then, with an opening of the compasses equal to one of the sides, draw an arc from one extremity of said line, and then from the other extremity draw another arc of the extent of the other contiguous side, which cuts the first arc, from which intersection of the two arcs draw right lines to each end of the diagonal; and for the two remaining sides prosecute the same method, and you shall have the figure required.

<i>Ch. l.</i>	
8.70	length.
8.70	breadth.
<hr/>	
60900	
6960	
<hr/>	
7.56900	<i>A. r. f.</i>
4	<i>Ans. 7 2 15</i>
<hr/>	
2.37600	
40	
<hr/>	
15.04000	
<hr/>	

N. B.

N. B. After multiplying the length by the breadth, instead of dividing by 10, the usual way, although tedious, I cut off a figure more than the four decimals, which is much more expeditious and equally exact.

P R O P. II.

What is the content of an oblong field, whose sides are exactly parallel to each other, as fig. 2. the length whereof is 15 ch. and 80 links, and breadth 9 chains 50 links?

This may be protracted precisely as the former.

$$\begin{array}{r}
 \text{Ch. l.} \\
 15.80 \\
 9.50 \\
 \hline
 79000 \\
 14220 \\
 \hline
 \text{Acres } 15.01000
 \end{array}$$

P R O P. III.

What is the content of a triangular field, as fig. 5. whole base is 20 ch. 75 l. and length of its perpendicular is 16 ch. 94 links?

Ch.

$$\begin{array}{r}
 \text{Ch. l.} \\
 20.75 \\
 8.47 = \frac{1}{2} \text{ perpendicular.} \\
 \hline
 14525 \\
 8300 \\
 16600 \\
 \hline
 17.57525 \\
 4 \\
 \hline
 2.30100 \\
 40 \\
 \hline
 12.04000
 \end{array}
 \qquad
 \begin{array}{r}
 A. \quad r. \quad f. \\
 \text{Ans. } 17 \quad 2 \quad 12
 \end{array}$$

# P R O P. IV.

To make a triangle that shall contain any number of acres, being confined to a certain base.

**RULE.** Bring the acres to falls, divide them by half the base, and the quotient is the perpendicular height of the triangle.

What will be the perpendicular of a triangle that shall contain 40 acres of ground, if the length of the base be 24 ch. 48 links?

$$\begin{array}{r}
 40 \\
 4 \\
 \hline
 160 \\
 40 \\
 \hline
 \text{Ch. l.} \quad \text{Ch. l.} \\
 \frac{1}{2} \text{ of } 24.48 = 12.24 \quad 6400.00 \quad (522.875 \text{ perpendicular.}) \\
 6120 \\
 \hline
 2800 \\
 2448 \\
 \hline
 3520 \\
 2448 \\
 \hline
 10720 \\
 9792 \\
 \hline
 9280 \\
 8568 \\
 \hline
 7120 \\
 6120 \\
 \hline
 1000 \\
 \hline
 \end{array}$$

## P R O P. V.

What is the content of a field, whose length is 12 ch. 50 l. and breadth at one end is 9 ch. 60 l. and the breadth of the other 7 ch. 84 links?

The common but erroneous method is, to take  $\frac{1}{2}$  of the two breadths when added together; but the true way is, to find out the mean proportional between the numbers that are the breadths; how-

† Y ever,

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ever, if it is necessary to take the breadths in 5, 6, or more places, if you add these breadths together, and divide them by the number of times they were taken, you will have the mean breadth, as near as can be found by any rule yet invented.

The true way.

<i>Ch. 1.</i>	<i>Ch. 1.</i>
12.55	9.60
8.67	7.84
<hr/>	<hr/>
8785	3840
7530	7680
10040	6720
<hr/>	<hr/>
10.88085	8)75.2640(8.67 mean propor-
4	64
<hr/>	<hr/>
3.52347	166)1126
40	996
<hr/>	<hr/>
20.93600	1727)13040
36	12080
<hr/>	<hr/>
561600	151
280800	<hr/>
<hr/>	
33.69600	
<hr/>	

The false way.

<i>Ch. 1.</i>
9.60
7.84
<hr/>
2)1744
<hr/>
8.72 mean breadth.

*Ans.*

<i>Ans.</i>	<i>Acres</i>	<i>r.</i>	<i>f.</i>	<i>ells.</i>	
	10	3	30	35	false way.
	10	3	20	33	true way.
<hr/>					
	10		2		difference.

P R O P. VI.

What is the content of an irregular field having 5 sides, as fig. 7. ?

**RULE.** Measure on the ground two diagonals, as you see in said figure, which will divide it into 3 triangles ; then measure all the sides, which will enable you to protract it upon paper ; and having drawn three perpendiculars ; suppose the 1st perpendicular 8 ch. 46 l. and the base on which it falls 7 ch. 38 l. and the 2d 6 ch. 88 l. and its base 4 ch. 90 l. and the 3d 9 ch. 40 l. and its base 7 ch. 20 l. *Quer.* the content of each triangle ?

<i>Ch. l.</i>	<i>Ch. l.</i>
1st base 7.38	2d base 4.90
$\frac{1}{2}$ of its perpen. = 4.23	$\frac{1}{2}$ of its perpen. = 3.44
<hr/>	<hr/>
2214	1960
1476	1960
2952	1470
<hr/>	<hr/>
3.12174	1.68560
<hr/>	<hr/>

Ch. 1.

3d base 7.20

 $\frac{1}{2}$  of its perpen.=4.70

---

50400

2880

---

3.38400

---

Content of the 1st triangle=3.12174

2d triangle=1.68560

3d triangle=3.38400

---

8.19134

4

---

.76536

40

---

30.61440

36

---

368640

184320

---

22.11840

---

A.	r.	f.	e.
Ans. 8	0	30	22

We have proceeded thus far upon a supposition, that all the figures referred to, may be measured with the chain within the field upon the ground; but as some cases may happen, such as in measuring a wood, a lake, or field of standing corn, as fig. 3. in that event the angles must be taken with a theodolite, and the four sides measured with a chain, and each of these must be marked in a field-book, as you see below.

Station		<i>Angles</i>	<i>Dist.</i>
1		60.00	4.15
2		120.00	4.15
3		60.00	4.15
4		120.00	4.15

After protracting said field according to this field-book, draw a perpendicular from any one of the angles to the sides opposite to said angle, and the length thereof is found to be 6 ch. 74 l. *Quar.* the content thereof?

**R U L E.** Multiply your perpendicular by the length of any one of the sides, which are all equal, and you have the content required.

*Ch. 1.*

6.74 the perpendicular.

4.15 one of the sides.

$$\begin{array}{r}
 3370 \\
 674 \\
 \hline
 2696 \\
 2.79710 \\
 4 \\
 \hline
 3.18840 \\
 40 \\
 \hline
 7.53600 \\
 36 \\
 \hline
 321600 \\
 190800 \\
 \hline
 16.29600
 \end{array}
 \begin{array}{l}
 A. r. f. c. \\
 Ans. 2 \quad 3 \quad 7 \quad 19
 \end{array}$$

PROP. VII.

What is the content of a circular field, as fig. 9. whose circumference is 16 ch. 33<sup>1</sup>/<sub>2</sub>?

*Ch.*

$$\begin{array}{r}
 \text{Ch. 1.} \\
 16.33 \\
 16.33 \\
 \hline
 4899 \\
 4899 \\
 9798 \\
 1633 \\
 \hline
 266.6689 \\
 .07957 \\
 \hline
 18666823 \\
 13333445 \\
 24000201 \\
 18666823 \\
 \hline
 2,1218844373 \\
 4 \\
 \hline
 4875377492 \\
 40 \\
 \hline
 19.5015999680 \\
 36 \\
 \hline
 30090598080 \\
 15045299040 \\
 \hline
 18.0543588480
 \end{array}
 \begin{array}{l}
 \text{A. r. f. c.} \\
 \text{Ans. 2 } 0 \ 19 \ 18
 \end{array}$$

Note, If any of the sides of the field is not straight, or is bounded with a rivulet; in that case, it will be necessary to use an offset-staff, the length of which is 7 feet 4 $\frac{1}{2}$  inches, which must be divided into 16 links, each 8.88 inches, with the one end like a square; and as you go along, take the offsets where-ever you observe a curve or an angle,

angle, remembering always to mark in your field-book the place of distance where you take said offset, and the number of links your offset amounts to, which you must mark opposite to the distance.

## P R O P. VIII.

There is a field in form of figure 2. which when measured we find to be 21 ch. 20 l. in length, and 7 ch. 8 l. in breadth; and its content is 15 acres: which field five gardeners want to be divided among them, in the manner following, viz. the 1st wanted one acre, the 2d two acres, the 3d three acres, the 4th four acres, and the 5th five acres. *Quer.* How much must be cut off from said field, so that each may have the quantity of ground as above?

*C. c. l.*  
Say 1 : 10 :: 7.08

$$\begin{array}{r}
 1 \\
 \hline
 7.08 \overline{) 10.00} \text{ (1.41} \\
 \underline{7.08} \\
 2920 \\
 \underline{2832} \\
 880 \\
 \underline{708} \\
 (172)
 \end{array}$$

	<i>Acres</i>	<i>Ch. l.</i>
<i>Ans.</i> 1st,	1 gets	1.41 long.
2d,	2	2.82
3d,	3	4.23
4th,	4	5.64
5th,	5	7.05
	<b>Proof</b>	<b>21.15</b>

P R O P.

PROP. IX.

There is an oval field, as fig. 10. whose longest diameter is 15 ch. 50 l. and its shortest diameter is 11 ch. 40 l. *Quær.* the content thereof?

**RULE.** Multiply the product of the two diameters by .7854,

$$\begin{array}{r}
 \text{Ch. l.} \\
 15.60 \\
 11.40 \\
 \hline
 62400 \\
 1560 \\
 1560 \\
 \hline
 177.8400 \\
 .7854 \\
 \hline
 7113600 \\
 8892000 \\
 14227200 \\
 12448800 \\
 \hline
 13.967553600 \\
 4 \\
 \hline
 3.870214400 \\
 40 \\
 \hline
 34.808576000 \\
 36 \\
 \hline
 4851456000 \\
 2425728000 \\
 \hline
 29.108736000
 \end{array}$$

*A. r. f. i.*  
*Ans.* 13 3 34 29

## OF GAUGING.

## SECT. IV. CHAP. I. PROP. I.

**T**O find the content in ale, wine, or corn gallons English measure, or in Scots pints, of a square ton or vessel.

**RULE.** Multiply the length or breadth in inches by itself, and the product is the area or content at one inch deep; which, multiplied by the height or depth, gives the solid content in inches, which to reduce to

$$\left. \begin{array}{l} \text{Ale-gall.} \\ \text{Wine-gall.} \\ \text{Corn-gall.} \\ \text{Scots pints,} \end{array} \right\} \text{Divide by } \left\{ \begin{array}{l} 282 \\ 231 \\ 268.8 \\ 102.3 \end{array} \right.$$

**EXAMPLE.** Suppose the side of a square vessel 40 2 inches, and height 10.3; how many gallons of ale, wine, or corn, or Scots pints doth it contain?

$40.2 \times 40.2 = 1616.04$ , which multiplied by the height 10.3 gives  $16645.212 \div 282 = 59.024$  fere ale gallons; which you may also reduce to wine or corn gallons or Scots pints, by using the above divisors.

## P R O P. II.

To find the contents in gallons of a vessel in form of a right-angled parallelogram.

**RULE.** Multiply the length by the breadth, and that product by the depth for the solid content

tent in inches, which reduce to gallons as before.

EXAMPLE. Suppose the length 60 inches, breadth 40, and depth 18; what is the content in gallons?

$60 \times 40 = 2400 \times 18 = 43200 \div 282 = 153.1872$  ale-gallons.

### PROP. III.

To find the content in ale, &c. gallons of a vessel of a triangular form.

RULE. Find the area of the base, which multiplied by the height gives the solid content in inches, and this divided by 282, &c. gives the content in gallons.

EXAMPLE. Suppose the length of the base of any triangular vessel be 25 inches, the perpendicular breadth 15, and the depth 12; what is the content in gallons?

$25 \times 7.55 = 187.6 \times 12 = 2250$  solid inches, which divided by 282 gives 7.9785 ale-gallons: and so on for wine, or corn gallons, and Scots pints.

Thus may any other figure contained in the preceding plate be measured, and its content found.

### PROP. IV.

To find the content in gallons &c. of any close cask.

1. If the staves of the cask have a great curve;

RULE. To twice the square of the greatest or bung diameter add the square of the lesser, and multiply that sum by the height of the cask,  
which

which product divided by 3.8197 quotes solid inches, and this divided by 282 gives gallons.

**EXAMPLE.** Suppose the bung diameter 20 inches, the lesser 16, and height of the cask 31; what is the content in gallons?

$$20 \times 20 = 400 \times 2 = 800$$

$$16 \times 16 = 256$$

$$\begin{array}{r} 1056 \times 31 = 32736 \div 3.8197 = \\ 8750.306 \div 282 = 30.39 \text{ ale-gallons.} \end{array}$$

2. If the staves are pretty straight from the bung to the head;

**RULE.** Add to the squares of the greater and lesser diameters their product, which multiplied by the height, and this product divided by 3.8197, quotes solid inches, which divided by 282 gives the content in gallons.

$$20 \times 20 = 400$$

$$16 \times 16 = 256$$

$$20 \times 16 = 320$$

$$\begin{array}{r} 976 \times 31 = 30256 \div 3.8197 = 7921.04 \div \\ 282 = 28.088 \text{ gallons of ale.} \end{array}$$

But such casks as these ought to be reduced to a cylinder, by taking a mean diemeter; for which observe the following rule. Multiply the difference between the head and bung diameter, by .7, .65, .6; or .55, according as the staves are more or less arching, which product add to the head-diameter, the sum is the mean diameter, and the cask is thereby reduced to a cylinder.

## GAUGING of MALT.

**A**CCORDING to an act of an English parliament, *anno* 1697, every round bushel with a plain and even bottom,  $18\frac{1}{2}$  inches wide throughout, and 8 inches deep, should be esteemed a legal Winchester bushel: now, such a vessel will contain 2150.42 cubic inches; for  $18.5 \times 15.5 = 342.25$ , which multiplied by .7854, gives 268.80315, and this last multiplied by the height 8, produces 215242. And therefore to find the number of bushels contained in any vessel, first find the solidity in inches, according to the form of the vessel, and divide by 2150.42 for the answer. If the malt be lying on the floor, in order to know the true depth, you must take the depth in several (suppose 6, 7, 8, or more) places, the sum of which divided by the number of places you took the depth in, quotes the mean depth.


**EXAMPLE.** Suppose a quantity of malt lying on the floor, in form of a rectangular parallelogram, length 160 inches, and breadth 100 inches. What is the number of bushels contained in it?

Suppose 1	5.5
2	6.
3 depth to be	4.8
4	5.9
5	4.8
6	6.1
7	5.7

38.8, which

divided by 7, the number of places, the quot 5.543 is the mean depth. Then  $160 \times 100 = 16000$ , and  $16000 \times 5.543 = 88688$  cubic inches. which divided by 2150.42, quotes 41.242 bushels for the answer.





## A P P E N D I X.

**I**N this Appendix, we do not propose to follow the exact order of the rules of Arithmetic and Mensuration, as laid down in the preceding part of this work, in order to render the resolution of the several questions somewhat more difficult; and let it suffice to observe, that the following questions have their respective answers placed separately and at some distance, precisely in the same order with the questions themselves.

*Question 1.* A father was 21 years 10 months and 6 days old, when his eldest son was born, and is now 68 years 1 month 20 days; how old is the son, accounting 30 days to the month, and 12 months to the year?

*Qu. 2.* If a round cistern be 26.3 inches diameter, and 52.5 inches deep, how many inches diameter must a cistern be to hold twice the quantity, the depth being the same?

*Qu. 3.* A father gave his daughter for her portion 24 boxes, and in each box were 16 lesser boxes, in each lesser box were 12 purses, and in each purse were 16 lesser purses, in each of which last were 4 d. Scots: what was her portion?

*Qu. 4.* A maltster had a kiln, that is 16 feet 6 inches square; but he intends to build a new one, that will dry three times as much at one time; I demand the square of the new one?

*Qu. 5.* What number divided by 3, 5, 7, and 12, will have no remainder?

*Qu.* 6. There is a stone 20 inches long, 15 broad, and 8 thick, which weighs 217 lb. ; I demand the length, breadth, and thickness of another of the same kind and shape, which will weigh 1000 lb. ?

*Qu.* 7. How many tuns burthen is that ship of, which can carry 11000 l. Sterling, when converted into Scots halfpence, each of which being  $\frac{1}{4}$  of an oz. Avoirdupoise ?

*Qu.* 8. The axis of a globe is 27.5 inches ; I demand the content, superficial and solid ?

*Qu.* 9. I went to a market with 150 l. Sterling, and a horse which cost me 5 guineas and an half : I sold the horse for 7 guineas, 10 s. ; I bought linen cloth to the value of 15 l. 12 s. whereof the seller discounted me a crown ; I bought a horse for 4 l. 14 s. 6 d. and spent of charges before I returned 12 s. 4 d. ; as I was coming home, I had the misfortune to drop a purse of 50 guineas. How much money remained ?

*Qu.* 10. What is the area of a semicircle whose diameter is 12.5 inches ?

*Qu.* 11. How many square stones of 10 inches, and  $1\frac{1}{2}$  inch thick, will pave a floor, which is 5 yards long and half as broad ?

*Qu.* 12. There is a board 16 inches long, and 9 inches broad ; how much must be cut from the length and breadth, so that a foot superficial may remain ?

*Qu.* 13. Add  $\frac{2}{7}$  l. to  $\frac{3}{4}$  s. and what is the value thereof ?

*Qu.* 14. If a man gains 6 d. per day, what will he gain in one year, including Sabbaths ?

*Qu.* 15. If a stone is 18 inches long, and its breadth  $\frac{2}{3}$  less than its length, and its thickness  $\frac{1}{3}$  less than its breadth ; how many inches must be cut off from its length, breadth, and thickness, so that no more than a solid foot may remain ?

*Qu.* 16. A has  $\frac{7}{10}$  of a ship, and B has  $\frac{2}{3}$  of the

the same; what is the difference of their shares;

*Qu.* 17. A set of boon companions dining at an inn, their reckoning came to 175 shillings; but before the bill was paid off, 2 of them flunk away, and then the club of those that remained came to 10 shillings a-man more: how many were there in company?

*Qu.* 18. If the freight of a ship be 8972 l. what mult A B get for  $\frac{7}{11}$  parts thereof?

*Qu.* 19. A company of men drank at an inn, till the reckoning came to 17 s.  $6\frac{1}{4}$  d. how many were in company, and what did each person pay?

*Qu.* 20. There is a room in form of a long square, whose length is 20 feet and breadth 15: how many yards of 3 quarter broad cloth will be sufficient to hang the same, its height being 7 feet?

*Qu.* 21. What number multiplied by  $5\frac{1}{2}$  produces 112 $\frac{3}{7}$ ?

*Qu.* 22. If any one thing cost  $\frac{5}{8}$  of a farthing, what is the value of 25 $\frac{1}{3}$  such things, at the same rate?

*Qu.* 23. What is the interest of 856 l. 18 s. 8 d. for 2 $\frac{1}{2}$  years at 4 per cent.?

*Qu.* 24. If of cloth that is  $1\frac{1}{4}$  yard broad, 2 $\frac{3}{4}$  yards will make a coat, how much in length of another cloth which is  $\frac{3}{4}$  of a yard in breadth, will make a coat of the same dimensions?

*Qu.* 25. If 6 d. gain  $1\frac{1}{3}$  far. how much per cent. is gained at that rate?

*Qu.* 26. If 4 $\frac{1}{3}$  lb. cost 9 $\frac{1}{3}$  d. how many lb. may be bought for  $\frac{2}{3}$  s. at the same rate?

*Qu.* 27. A B merchant in Aberdeen shipped on board the Nancy, for Jamaica. 500 yards of linnen at 2 s. 8 d. per yard, and 60 dozen of stockings at 3 s. 5 d. per pair, and 200 yards of check at 1 s. 10 d. per yard, for which he had in return 10 hogshheads of sugar, each weighing 12 Cwt. at

30 s. per Cwt. and 300 gallons of rum at 4 s. 8 d. per gallon: I demand how much he gained upon the whole?

*Qu.* 28. In a plank of mahogany 16 f. 6 inches long, 2 f. 4 inches broad, and 11 inches thick, how many feet sawyers measure, allowing  $\frac{1}{8}$  of an inch for the draught of the saw?

*Qu.* 29. What is  $\frac{6}{19}$  of 12 l. 10 s. 8 d.  $3\frac{1}{2}$  f.?

*Qu.* 30. If  $\frac{1}{3}$  d. buy  $2\frac{1}{8}$  oz. how much will 4 l. buy?

*Qu.* 31. If 8 men do a piece of work in 6 days, in what time will they do  $16\frac{1}{2}$  times as much?

*Qu.* 32. What will 54 l. amount to, being forborne 15 years, at 6 per cent per annum, simple interest?

*Qu.* 33. A farmer mixes 8 pecks of wheat at 16 d. per peck, with 9 pecks at 18 d. with 12 pecks at 17 d. what is a peck of the mixture worth?

*Qu.* 34. In 36 Cwt. 1 qr. 12 lb. gross, how much nett weight, tare at 8 lb. per 112 lb. and trett 4 lb. per 104?

*Qu.* 35. In what time will B perform a piece of work, when A alone can do it in  $17\frac{1}{7}$  days, and A and B together in 15 days?

*Qu.* 36. A certain person lent 300 l. and at the end of 3 years received 357.3048 l. *Quer.* at what rate accumulated interest the money was lent?

*Qu.* 37. In what time will a pendulum 130 inches long make one vibration?—**RULE.** Multiply the length in inches by the decimal .025553, and the square root of the product is the time in seconds.

*Qu.* 38. To find two numbers in the proportion of 2 to 3, whose product, if they be multiplied into one another, shall be 54?

*Qu.* 39. How many vibrations will a pendulum 5 inches long make in one hour, or 36000 seconds?

*Qu.*

Qu. 40. In what time will a pendulum 5 inches long make 10072.74 vibrations ?

Qu. 41. Between seeing the lightning and hearing the thunder, were measured 28 vibrations of a pendulum 28 inches long, I demand the observer's distance from the thunder ?

Qu. 42. A ship at sea sees a privateer fire a gun, the interval of the sound and smoke was measured by 112 vibrations of a pendulum 20.7 inches long ; I demand the observer's distance from said privateer ?

Qu. 43. What is an estate of 200 l. *per annum* to continue for ever, worth in ready money, allowing the purchaser 5 l. *per cent. per annum*, compound interest ?

Qu. 44. A gentleman wants a piece of ground paved before his door with stones 3 feet long and 2 feet broad ; the ground is 4 yards broad and 30 yards long ; how many stones will serve ?

Qu. 45. Suppose 4000 l. were proposed to be laid out in the purchase of a freehold estate ; what annual rent would it buy, allowing the purchaser 5 *per cent. per annum* ?

Qu. 46. How much printed paper will line a room that is 70 yards in circumference and 6 yards high, if the paper be 3 quarters broad ?

Qu. 47. Suppose a freehold estate of 200 l. *per annum* cost 4000 l. what rate of interest *per cent.* is allowed the purchaser ?

Qu. 48. What number is that, from which if the square of 14 is deducted, and to the remainder the square of 12 is added, the sum will be 250 ?

Qu. 49. A and B traded together ; A put in 320 l. for five months, and B 460 l. for three months, and they gained 100 l. what must each receive ?

Qu. 50. How many yards of cloth, at 17 s. 6 d.

6 d. per yard, can I have for 13 Cwt. 2 qrs of wool, at 14 d. per lb. ?

*Qu.* 51. What number added to the cube of 21, will make the sum equal to 113 times 147 ?

*Qu.* 52. What number taken from the square of 54 will leave 19 times 46 ?

*Qu.* 53. If I buy a yard of cloth for 14 s. 6 d. and sell it again for 16 s. 9 d. what do I gain *per cent.* ?

*Qu.* 54. If  $\frac{3}{4}$  of an ounce cost  $\frac{1}{8}$  of a shilling, what will  $\frac{5}{8}$  of a lb. cost ?

*Qu.* 55. A young man received 210 l. which was  $\frac{2}{3}$  of his elder brother's portion ; now three times the elder brother's portion was half of the father's estate : I demand how much the estate amounted to ?

*Qu.* 56. If an officer's pay be 48.5 l. *per annum*, what must he receive for 232 days ?

*Qu.* 57. What number is it, to which if you add  $7\frac{2}{3}$ , the whole will be  $12\frac{1}{4}$  ?

*Qu.* 58. A certain usurer put out 75 l. for 12 months, and received for principal and interest 81 l. I demand at what rate *per cent.* he received interest ?

*Qu.* 59. At what rate *per cent.* will 956 l. amount to 1314 l. 10 s. in  $7\frac{1}{2}$  years, at simple interest ?

*Qu.* 60. If for 1 l. 4 s. I have 1200 lb. weight carried 36 miles, how many lb. weight can I have carried 24 miles for the same money ?

*Qu.* 61. What number is it, which being multiplied by  $\frac{2}{3}$  will produce  $\frac{1}{4}$  ?

*Qu.* 62. A man dies, and leaves 120 l. to be given to 3 persons, *viz.* A, B, and C ; to A a share unknown ; to B twice as much as to A, and C as much as A and B ; what is each person's share ?

*Qu.* 63. T has 24 cows worth 72 s. each, and B 7 horses worth 13 l. a-piece ; how much will make good the difference, in case they interchange their said drove of cattle ?

*Qu.*

*Qu.* 64. A piece of wainscot is 8 feet  $6\frac{1}{2}$  inches long, and 2 feet  $9\frac{1}{2}$  inches broad, what is the superficial content?

*Qu.* 65. A merchant in Amsterdam is indebted to another in London, for 642 l. and would pay it in Spanish guilders, at 2 s. per piece; how many must the English merchant receive?

*Qu.* 66. The lesser of two numbers is 187, their difference 34, the square of their product is required?

*Qu.* 67. A butcher sends his man with 216 l. to a fair to buy cattle, oxen at 11 l. cows at 40 s. colts at 11. 5 s. and hogs at 1 l. 15 s. per piece, and of each a like number; how many of each sort did he buy?

*Qu.* 68. What number added to  $11\frac{1}{2}$  will produce  $36\frac{1}{2}\frac{1}{8}$ ?

*Qu.* 69. What is the value of 179 hogshheads of tobacco, each weighing 13 Cwt. at 2 l. 7 s. 1 d. per Cwt.?

*Qu.* 70. There is in three bags the sum of 1463 l. viz. in the first bag 461 l. in the second 531 l. what was in the third bag?

*Qu.* 71. How many lb. of sugar at  $4\frac{1}{2}$  d. per lb. must be given in barter for 60 gros of incle, at 8 s. 8 d. per gros?

*Qu.* 72. Miss Kitty told her sister Charlotte, whose father had before left them twelve thousand twelve hundred pounds a-piece, that their grandmother by will had raised her fortune to fifteen thousand pounds, and had made her own twenty thousand; what did the old lady leave between them?

*Qu.* 73. If I buy yarn for 9 d. the lb. and sell it again for  $13\frac{1}{2}$  d. per lb. what is the gain *per cent.*?

*Qu.* 74. Two persons, A and B, owe several debts; the lesser debt, being that of A, is 2173 l. the difference is 371 l. what is B's debt?

*Qu.* 75. An old lady being asked how old she was, to avoid a direct answer, said, I have nine children,

children, and there are 3 years between the birth of each of them; the eldest was born when I was 19 years old, which is now exactly the age of the youngest; how old was the lady?

*Qu.* 76. What number added to the 43d part of 4429 will make the sum 240?

*Qu.* 77. My purse and money, says Dick, are worth 12 s. 8 d. but the money is worth 7 of the purse; pray what was there in it?

*Qu.* 78. I bought a cask of wine for 62 l. 8 s. how many gallons were therein, when a gallon was valued at 5 s. 4 d.?

*Qu.* 79. A owes B 296 l. 17 s. but he compounds for 7 s. 6 d. in the pound; what must B receive for his debt?

*Qu.* 80. How many dozens of stockings, at 11 groats per pair, may I buy for 190 l. 12 s.?

*Qu.* 81. If the content of a globe, cylinder, cone, or such like, be 15625 solid inches; what will be the side of a cube equal in capacity thereto?

*Qu.* 82. I want two mean proportionals between 6 and 162?

*Qu.* If  $\frac{1}{3}$  of a ship be worth 37.40 l. what is the worth of the whole?

*Qu.* 84. A person said he had 20 children, and that it happened there was a year and a half between each of their ages; his eldest was born when he was 24 years old, and the age of the youngest is now one and twenty; what was the father's age?

*Qu.* 85. What is the mean proportional between 3 and 12?

*Qu.* 86. If the content of a circle is 160, what is the side of a square equal to it? N. B. The square root of the content of any given superficies is the square equal sought.

*Qu.* 87. What will be the pay of 540 men, at 1 l. 5 s. 6 d. per man?

*Qu.* 88. If 3 men can do a piece of work in  $4\frac{1}{2}$  hours,

hours, in how many hours will 10 men do the same work?

*Qu.* 89. If the penny-loaf weigh 7 ounces, when a bushel of wheat costs 5 s. 6 d. what is the bushel worth, when the penny-loaf weighs but  $2\frac{1}{2}$  oz.?

*Qu.* 90. How many yards of canvass that is  $1\frac{1}{4}$  yard wide, will be sufficient to line 20 yards of say that is  $\frac{1}{4}$  of a yard wide?

*Qu.* 91. In 179 bitts of Jamaica at  $7\frac{1}{2}$  each, how many pounds Sterling?

*Qu.* 92. How many Spanish patacoons at 4 s. 8 d. each, must I receive for 12 l. 12 s. Sterling?

*Qu.* 93. A merchant bought 84 pieces of cloth for 537 l. 12 s. which was at the rate of 5 s. 4 d. per yard; I demand how many yards were in all, and how many ells English were in a piece of the same?

*Qu.* 94. A merchant bought 242 yards of cloth, for 254 l. 10 s. for 86 yards of the same he paid after the rate of 21 s. 4 d. per yard; I demand how much he gave for the yard of the remainder?

*Qu.* 95. If I lend my friend 240 l. for 5 months, how long must I keep 400 l. of his, to requite myself?

*Qu.* 96. A garrison consisting of 1764 men, being besieged, had only provisions for 12 days; but it being necessary that they should hold out 3 weeks, how many men must be sent out?

*Qu.* 97. How long shall I be in laying up 1000 l. Sterling, if I put by 2 l. 10 s. 6 d. per week?

*Qu.* 98. If at 5 s. per yard, I gain 8 l. *per cent.* on a quantity of cloth, what shall I gain *per cent.* if I sell the yard at 6 s. 3 d.?

*Qu.* 99. If the area of a circle is 750, what is the side of a square equal to it?

*Qu.* 100. If I give 1 s. 1 d. for  $3\frac{1}{2}$  lb. of cheese, what will 1 Cwt. give?

The

The following questions have their answers immediately annexed to them.

*Qu.* 101. What is the value of 14 barrels of soap, at  $4\frac{1}{2}$  d. per lb. each barrel containing 254 lb. ? *Ans.* 66 l. 13 s. 6 d.

*Qu.* 102. Two persons, A and B, owe several debts ; the lesser debt, being that of A, is 2173 l. the difference is 371 l. what is the debt of B ? *Ans.* 2544 l.

*Qu.* 103. What number deducted from the 26th part of 2262 will leave the 87th part of the same ? *Ans.* 61.

*Qu.* 104. A gentleman went to sea at 17 years of age ; 8 years after he had a son who lived 46 years, and died before his father ; after whom the father lived twice 20 years, and then died also ; *Quer.* the age of the father when he died ? *Ans.* 111 years.

*Qu.* 105. C hath candles at 6 s. per dozen ready money, but in barter will have 6 s. 6 d. per doz. D hath cotton at 9 d. per lb. ready money. I demand what price the cotton must be at in barter, also how much cotton must be bartered for 100 doz. of candles ? *Ans.* 9 d. 3 f. per lb. and 7 Cwt. 16 lb. must be given for 100 dozen of candles.

*Qu.* 106. The sum of 2 numbers is 360, the less is 114, what is their difference, product, and larger quot ? *Ans.* 132 diff. 28044 prod.  $2\frac{3}{4}$  quot.

*Qu.* 107. A brigade of horse consisting of 384 men is to be formed into a square body, having 32 in front, how many ranks will there be ? *Ans.* 12.

*Qu.* 108. If a clerk's salary be 73 l. a-year, what is that per day ? *Ans.* 4 s.

*Qu.* 109. A hath an estate of 53 l. *per annum*, and payeth 5 s. 10 d. to the subsidy ; what must B pay, whose estate is worth 100 l. *per annum* ? *Ans.*  $11\frac{4}{3}$  s.

*Qu.*

*Qu.* 110. If I buy 100 yards of riband at 3 yards for a shilling, and 100 more at 2 yards for a shilling, and sell it again at the rate of 5 yards for 2 shillings, whether do I gain or lose, and how much? *Ans.* 3 s. 4 d.

*Qu.* 111. What number is that, from which if you take  $\frac{1}{4}$ , the remainder will be  $\frac{1}{8}$ ? *Ans.*  $\frac{1}{4}$ .

*Qu.* 112. What number is that which maketh 9 to be  $\frac{2}{3}$  of it? *Ans.*  $13\frac{1}{2}$ .

*Qu.* 113. The Spectator's club of fat people, though it consisted but of 15 persons, is said to weigh no less than 3 tons, how much at an equality was that per man? *Ans.* 4 Cwt.

*Qu.* 114. A owes B 395 l. 18 s. but compounds the whole debt for 100 l. 12 s. what is that per pound? *Ans.* 5 s. 1 d. nearly.

*Qu.* 115. What is the amount of 1000 l. for  $5\frac{1}{2}$  years, at  $4\frac{1}{4}$  per cent. simple interest? *Ans.* 1261 l. 5 s.

*Qu.* 116. Two men depart from one place; the one goes east, and the other west; the one goes 7 miles a-day, and the other 11 miles a-day; how far are they distant the 12th day after their departure? *Ans.* 216 miles.

*Qu.* 117. A merchant bought 8 tuns of wine, which having received damage, he sold again for 400 l. and 12 l. per cent. loss; I demand how much it cost per tun, and how he sold it per gallon, to lose after the said rate? *Ans.* 56 l. per tun prime cost, and at 3 s.  $11\frac{1}{4}$  d. per gallon.

*Qu.* 118. If 240 lb. of tobacco cost 13 l. what will be the price of 1 lb. so as to gain 15 l. 10 s. per cent.? *Ans.* 1 s. 3 d. per lb.

*Qu.* 119. If the beam of a balance is 63 inches long, and 84 lb. on one end weighs 112 lb. on the other, I demand the length of the arms of the said balance? *Ans.* 36 inches the one, and 27 the other.

*Qu.* 120. If I buy at 15 s. 10 d. and gain by the sale 25 l. *per cent.* how must I buy to gain by the same sale 35 l. *per cent.*? *Ans.* 14 s.  $7\frac{1}{2}$  d.

*Qu.* 121. What principal sum forborn 7 years at 5 *per cent. per annum*, simple interest, will amount to 200 l. 9 s. 6 d. at the 7 years end? *Ans.* 148 l. 10 s. principal sum.

*Qu.* 122. Supposing 3 to be  $\frac{1}{3}$  of 12, what would  $\frac{1}{4}$  of 20 be? *Ans.*  $3\frac{3}{4}$ .

*Qu.* 123. What is the value of  $\frac{1}{16}$  of an ounce, at  $3\frac{1}{2}$  d. per lb.? *Ans.*  $\frac{7}{16}$  farthing.

*Qu.* 124. I bought at one time  $19\frac{1}{7}$  lb. of a certain commodity at  $18\frac{1}{3}$  d. per lb. at another time I bought of the same commodity  $58\frac{4}{9}$  lb. at the rate of  $1\frac{1}{7}$  d. per ounce; what came each to at their respective prices; and which of them was the better bargain, and by how much per lb.?

*Ans.* The value of  $19\frac{1}{7}$  lb. at  $18\frac{1}{3}$  d. per lb. is 1 l. 9 s. 4 d.  $6\frac{2}{3}$  f. the value of  $58\frac{4}{9}$  lb. at  $1\frac{1}{7}$  d. per ounce, is 4 l. 9 s. 0 d.  $2\frac{8}{9}$  f. and the last is the cheapest bargain by  $\frac{4}{15}$  per lb.

*Qu.* 125. There are 3 numbers, 17, 19, and 48; I demand the difference between the sum of the squares of the first and the last, and the cube of the middlemost? *Ans.* 4266.

*Qu.* 126. A bath  $\frac{1}{2}$  of a ship, B  $\frac{1}{4}$ . C  $\frac{1}{8}$ , D  $\frac{1}{16}$ , the master clears 120 l. how much must each owner have? *Ans.* A 60 l. B 30 l. C 7 l. 10 s. D 2 l. 10 s.

*Qu.* 127. If I buy 1000 ell's Flemish of linen for 90 l. what may I sell it at per ell in London to gain 10 l. on the whole? *Ans.* 3 s. 4 d. per ell.

*Qu.* 128. What number is that, which being multiplied by 15, the product will be  $\frac{3}{4}$ ? *Ans.*  $\frac{1}{20}$ .

A N S W E R S.

- Ans.* 1.  $y. m. d.$  46 3 14, for  $68 \ 1 \ 20 - 21 \ 10 \ 6 = 46 \ 3 \ 14.$
- Ans.* 2. 37.19, for  $26.3 \times 26.3 = 691.69 \times 2 = 1383.38$ , the square root of which  $= 37.19$ , the diameter required.
- Ans.* 3. 1228 l. 16 s. for  $24 \times 16 \times 12 \times 16 \times 4 = 294912$  d. or 1228 l. 16 s.
- Ans.* 4. 28.57, for  $16.5 \times 16.5 = 272.25$ , the area of the old  $\times 3 = 716.75$ , whose square root is 28.57 = the side of the square.
- Ans.* 5. 1260, for  $5 \times 3 \times 7 \times 12 = 1260.$
- Ans.* 6. 33.28 inches length, 24.96 inches breadth, 13.312 inches thickness, for the cube of 20 = 8000 and  $217 : 8000 :: 1000 : 36870.645$ , whose cube root is 33.28 inches, and  $20 : 33.28 :: 15 : 24.96$   
 $20 : 33.28 :: 8 : 13.312$
- Ans.* 7. Tuns 36 6 2 12, for  $11000 \times 240 \text{ d.} = 2640000 \times 2 = 5280000 \div 4 = 1320000 \text{ oz.} \div 16, 28, 4$ , and  $20 \times 36 \text{ T. } 6 \text{ Cwt. } 2 \text{ qrs } 12 \text{ lb.}$
- Ans.* 8. F. 16.49, for  $3.1416 \times 27.5 = 86.39400$  circumference  $\times 27.5 = 2375.835.000 \div 6 = 397.9725 \times 27.5 = 10889.24375 \div 1728 = 6.3$  feet solid, and  $2375.8350 \div 144 = 16.49$  feet superficial.
- Ans.* 9. 84 l. 12 s. 2 d. for  $150 \text{ l. } + 7 \ 17 = 157 \ 17$ , and  $15 \text{ l. } 12 \text{ s. } - 5 \text{ s.} = 15 \text{ l. } 7 \text{ s. } + 4 \text{ l. } 15 \text{ s. } 6 \text{ d.} = 20 \text{ l. } 2 \text{ s. } 6 \text{ d. } + 12 \text{ s. } 4 \text{ d.} = 20 \text{ l. } 14 \text{ s. } 10 \text{ d. } + 5 \text{ l. } 10 \text{ s.} = 73 \ 4 \text{ l. } 10 \text{ s.}$  and  $157 \ 17 - 73 \ 4 \ 10 = 84 \text{ l. } 12 \text{ s. } 2 \text{ d.}$
- Ans.* 10. 61.359375, for  $12.5 \times 12.5 \times .3927$  (viz.  $\frac{1}{2}$  of .7854)  $= 61.359375.$
- Ans.* 11. 162, for  $36 \times 5 = 180 \times 90 = 16200 \div 100 = 162.$
- Ans.* 12. 0, for  $16 \times 9 = 144$  inches = a superficial foot.
- Ans.* 13.  $633 \frac{3}{4}$ , for  $\frac{3}{4}$  of  $\frac{1}{5} = \frac{3}{20} = \frac{3}{100} + \frac{3}{7} = \frac{20}{700} + \frac{3}{7} = \frac{20}{700} + \frac{300}{700} = \frac{320}{700} = \frac{32}{70} = \frac{16}{35}$  far.
- Ans.* 14.  $6 + \text{its } \frac{1}{2} \ 3 = 9$  and 5 (being the days above 360)  $\times 6 \text{ d.} = 2 \text{ s. } 6 \text{ d.}$  and the whole amount  $= 9 \text{ l. } 2 \text{ s. } 6 \text{ d.}$  — RULE. In cases of this nature add always to your pence, the  $\frac{1}{2}$  thereof give pounds, and add thereto the sum arising from the pence  $\times 5$ .
- Ans.* 15. 0, for  $18 \times 12 \times 8 = 1728$  inches, which makes a solid foot.
- Ans.* 16.  $\frac{1}{10}$ , for  $\frac{7}{10} - \frac{2}{5} = \frac{7}{10} - \frac{4}{10} = \frac{3}{10} = \frac{1}{10}.$

A a 2

*Ans.*

*Ans.* 17. 7, for  $175 \times 2 = 350 \div 10 = 35 + \frac{1}{4}$  of the square of  $2 = 36$ , whose square root is  $6 + \frac{1}{2}$  of  $2 = 7$ .

*Ans.* 18. 1847 l. 3 s.  $6\frac{1}{2}$  d. for  $34 : 8972 :: 7$ , and  $8972 \times 7 = 62804 \div 34 = 1847.177$ .

*Ans.* 19. 29 men, and each paid  $7\frac{3}{4}$  d. for 17 s.  $6\frac{3}{4}$  d. = 841 far. whose square root is 29.

*Ans.* 20. 72 yards 2 qrs 1 nail, for  $20 + 15 = 35 \times 2 = 70 \times 7 = 490$  square feet = 70560 square inches, and  $27 \times 36 = 972$  square inches in 1 yard, and  $70560 \div 97 = 72$  yards 2 qrs. 1 nail.

*Ans.* 21.  $20\frac{3}{4}$ , for  $5\frac{1}{2}(112\frac{3}{4}) = \frac{1}{2}^7 \frac{7}{7}(\frac{1}{7} \frac{7}{7} = 20\frac{3}{4}$ .

*Ans.* 22. 3 d.  $3\frac{1}{8}$  f. for  $\frac{7}{3} \times \frac{5}{8} = \frac{35}{24} = 15$  s. = 3 d.  $3\frac{1}{8}$  f.

*Ans.* 23. 85 l. 13 s.  $10\frac{3}{4}$  d. for  $4 \times 2\frac{1}{2} = 10$  (856 l. 18 s. 8 d.) 85 l. 13 s.  $10\frac{3}{4}$  d.

*Ans.* 24. 4 yards 2 qrs.  $1\frac{1}{2}$  nail, for  $1\frac{1}{4} : 2\frac{1}{2} :: \frac{3}{4} = \frac{5}{4} : \frac{1}{4} :: \frac{3}{4} = \frac{5}{4} \frac{5}{8}(\frac{2}{4} \frac{10}{8} = 4\frac{7}{8} = 4$  yards 2 qrs.  $1\frac{1}{2}$  nail.

*Ans.* 25. 5 l. for .5 : X.025 :: 100 &  $100 \times .025 = 2.500 \div .5 = 5$  l.

*Ans.* 26.  $1\frac{1}{2}$  lb. for  $9\frac{1}{3} : 4\frac{1}{3} :: \frac{2}{3}$  of  $\frac{1}{2} = \frac{2}{3} : \frac{2}{3} :: \frac{2}{3} : \frac{2}{3}$  Therefore  $\frac{2}{3} \times 2 = \frac{4}{3} \div 7 = \frac{6}{7} = 1\frac{1}{7}$ .

*Ans.* 27. 42 l. for 500 yards at 2 s. 8 d. = 66 l. 13 s. 4 d. + 60 dozen at 3 s. 5 d. per pair, 123 l. + 200 yards at 1 s. 10 d. = 18 l. 6 s. 8 d. whose sum = 208 l. and 10 hogheads each 12 Cwt. at 30 s. = 180 + 200 gallons at 4 s. 8 d. = 70 = 250 - 208 = 42.

*Ans.* 28. 385 feet, for  $166 \times 2 - 4 = 386 \times 10 = 385$ .

*Ans.* 29. 3 l. 19 s. 2 d.  $\frac{1}{9}\frac{6}{7}$  far. for 12 l. 10 s. 8 d.  $\frac{3}{5}$  far. reduced =  $\frac{60}{5} \frac{1}{7} \frac{6}{9} \times \frac{6}{19} = \frac{36}{95} \frac{6}{5} = 3800$ , when reduced = 3 l. 19 s. 2 d.  $\frac{1}{9}\frac{6}{7}$  far.

*Ans.* 30.  $427\frac{1}{2}$  lb. for  $\frac{1}{2} : 2\frac{3}{8} :: 4 = 7\frac{7}{8} : \frac{1}{8} :: \frac{4}{1} = 7\frac{7}{8}$  ( $\frac{7}{8} = (\frac{4}{8} \frac{7}{8} \frac{20}{8} = 6840 \div 16 = 427\frac{1}{2}$  lb.

*Ans.* 31. 99 days, for  $1 : 6 :: 16\frac{1}{2}$  and  $16\frac{1}{2} \times 6 = 99$  d.

*Ans.* 32. 102 l. 12 s. for  $100 : 6 :: 54 : 3$  l. 4 s. 9 d.

*Ans.* 33. D.  $17\frac{1}{2}$ , for 8 at 16 = 128, and 9 at 18 = 162, and 12 at 17 = 204 : and  $29 : 494 :: 1$ , then  $494 \times 1$  and  $29 = 17\frac{1}{2}$ .

*Ans.* 69.  $5478 \text{ l. } 2 \text{ s. } 11 \text{ d. for } 179 \times 13 = 2327 \times 2 = 4654 + 581 = 5235$   $15 + 9 = 24$   $13 = 27$   $11 = 22$   $5478 \text{ l. } 2 \text{ s. } 11 \text{ d.}$

*Ans.* 70.  $426 \text{ l. for } 461 + 581 = 1042$ , and  $1468 - 1042 = 426 \text{ l.}$

*Ans.* 71.  $1386\frac{2}{3} \text{ lb. for } 60 \text{ gross at } 8 \text{ s. } 8 \text{ d. is } 26 \text{ l. } \div 18 \text{ qrs. gives } 1386\frac{2}{3} \text{ lb.}$

*Ans.* 72.  $8600 \text{ l. for } 15000 - 13200 = 1800$ , and  $20000 - 13200 = 6800 + 1800 = 8600 \text{ l.}$

*Ans.* 73.  $50 \text{ l. for if } 9 \text{ d. yields } 13\frac{1}{2} \text{ d. } 100 \text{ will yield } 150$ , which is  $50 \text{ l. gained.}$

*Ans.* 74.  $2544 \text{ l. for } 2173 + 371 = 2544 \text{ l.}$

*Ans.* 75.  $62 \text{ years, for } 19 + 24 + 19 = 62.$

*Ans.* 76.  $137$ , for  $4429 \div 43 = 103$ , and  $240 - 103 = 137.$

*Ans.* 77.  $11 \text{ s. } 7 \text{ d. for } 12 \text{ } 8 \div 8 = 1$   $7$ , and  $12 \text{ } 8 - 1$   $7 = 11 \text{ s. } 7 \text{ d.}$

*Ans.* 78.  $234 \text{ gallons, for } 1.62 \text{ } 8 = 14976 \text{ d. } \div 64 = 234.$

*Ans.* 79.  $111 \text{ l. } 6 \text{ s. } 4\frac{1}{2} \text{ d. for } 7 \text{ s. } 6 \text{ d. } = \frac{3}{8} \text{ of a pound, and } \frac{3}{8} \text{ of } 296 \text{ l. } 17 \text{ s. } = 111 \text{ l. } 6 \text{ s. } 4\frac{1}{2} \text{ d.}$

*Ans.* 80.  $86 \text{ doz. } 7 \text{ pairs } \frac{2}{3}$ , for  $45744 \div 4 = 1039 \div 12 = 86 \text{ doz. } 7 \text{ pairs } \frac{2}{3}.$

*Ans.* 81.  $25 \text{ inches, for the cube root of } 15625 = 25.$

*Ans.* 82.  $18 \text{ and } 54$ , for  $162 \div 9 = 27$ , whose cube root is  $3 \times 6 = 18$  the lesser, and  $18 \times 3 = 54$  the greater.

*Ans.* 83.  $9973 \text{ l. } 6 \text{ s. } 8 \text{ d. for } 3:3740::8:9973 \text{ l. } 6 \text{ s. } 8 \text{ d.}$

*Ans.* 84.  $73\frac{1}{2} \text{ years, for } 24 + 19 + 1\frac{1}{2} + 21 = 73\frac{1}{2} \text{ years.}$

*Ans.* 85.  $6$ , for  $3 \times 12 = 36$ , whose square root is  $6.$

*Ans.* 86.  $12.64911$ , for the square root of  $160$  is  $12.64911.$

*Ans.* 87.  $688 \text{ l. } 10 \text{ s. for } 1:1.275::540:688 \text{ l. } 10 \text{ s.}$

*Ans.* 88.  $1\frac{7}{8} \text{ hour, for } 3:4.5::10:1\frac{7}{8}.$

*Ans.* 89.  $15 \text{ s. } 4\frac{4}{5} \text{ d. for } \frac{7}{8}:\frac{6}{8}::\frac{5}{8}:\frac{5}{8} \text{ } (\frac{9}{5})^4 = 184\frac{4}{5} \div 12 = 15 \text{ s. } 4\frac{4}{5} \text{ d.}$

*Ans.* 90.  $12 \text{ yards, for } \frac{3}{4}:\frac{20}{1}::\frac{5}{4}$ , and  $20 \times 3 = 60 \div 5 = 12 \text{ yards.}$

*Ans.* 91.  $5 \text{ l. } 11 \text{ s. } 10\frac{1}{2} \text{ d. for } 7 \text{ d. } 2 \text{ f. } \times 10 \times 10 + 7 \text{ d. } 2 \text{ f. } \times 70 + 7 \text{ d. } 2 \text{ f. } \times 9 = 5 \text{ l. } 11 \text{ s. } 10\frac{1}{2} \text{ d.}$

*Ans.* 92.  $54 \text{ patacoons, for } 12 \text{ l. } 12 \text{ s. } \times 20 \times 12 = 3024 \text{ d. which divided by } 56 = 54 \text{ patacoons.}$

*Ans.* 93.  $2016 \text{ yards in all, and } 19\frac{1}{2} \text{ English ells in } 1 \text{ piece,}$

1 piece, for 64 d. : 1 yard :: 129024 d. :  $2016 \times 4 = 8064 \div 84 = 96 \div 5 = 19\frac{1}{5}$ .

*Ans.* 94. 1 l. 0 s. 10 d. for 86 yards amount to 91 l. 14 s. 8 d. at 21 s. 4 d. per yard, and  $242 - 86 = 156$  yards, and  $254$  l. 10 s. — 91 l. 14 s. 8 d. = 162 l. 15 s. 4 d. so that  $156 : 39064 :: 1 : 250 \div 12 = 20$  s. 10 d.

*Ans.* 95. 3 months, for  $240 : 5 :: 400 : 3$ .

*Ans.* 96. 756 men, for  $12 : 1764 :: 21 : 1008$  men kept, and  $1764 - 1008 = 756$ .

*Ans.* 97. 5 years 23 weeks and 4 days, for 846 d. : 1 w. ::  $240000$  d. : 283 w. 4 d.

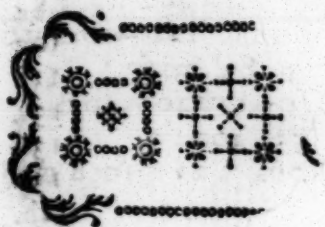
*Ans.* 98. 10 l. *per cent.* for 60 d. : 8 l. :: 75 d. : 10 l.

*Ans.* 99. 27.38612, for the square root of  $750 = 27.38612$ .

*Ans.* 100. 1 l. 14 s. 8 d. for  $3\frac{1}{2}$  lb. : 13 d. :: 112 : 1 l. 14 s. 8 d.

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